

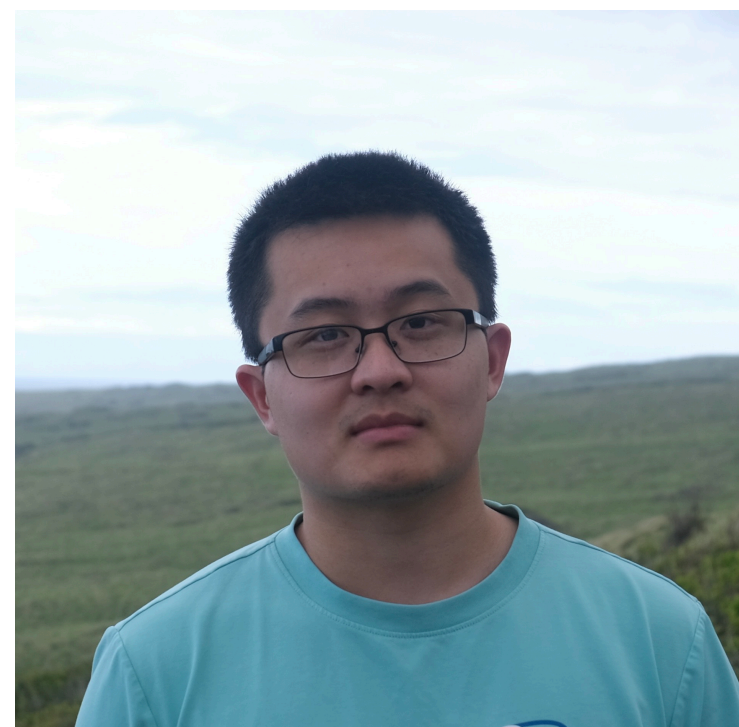
Mixed-State Long-Range Order and Criticality from Measurement and Feedback

Tsung-Cheng (Peter) Lu

(Perimeter Institute)

arXiv: 2303.15507 (PRX Quantum) **Lu**, Zhang, Vijay, Hsieh

Zhehao Zhang
(UCSB)



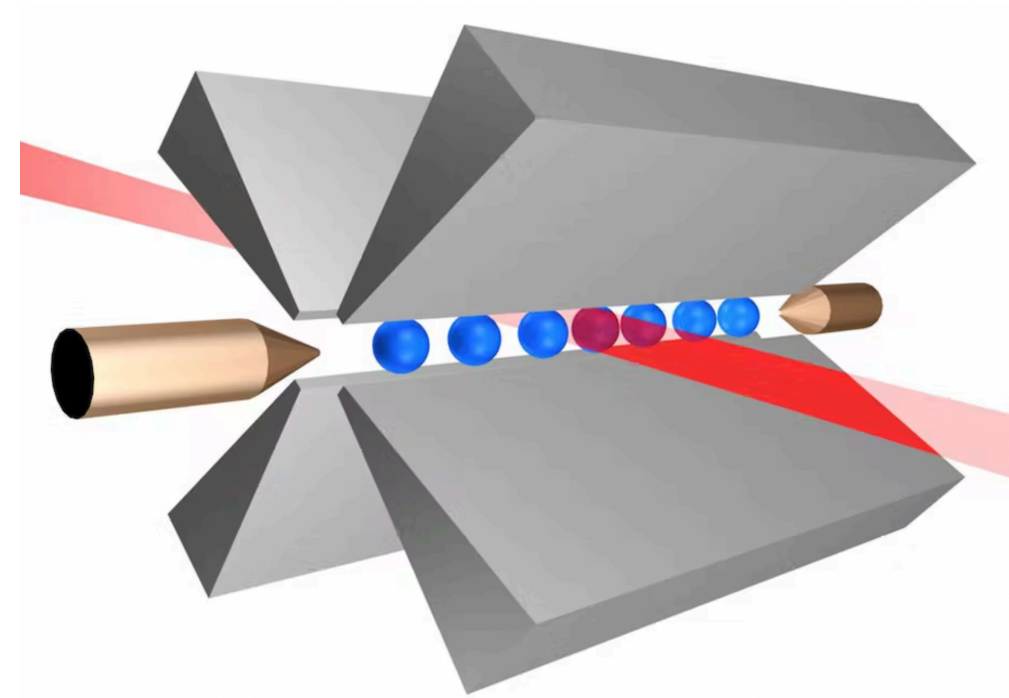
Sagar Vijay
(UCSB)



Tim Hsieh
(Perimeter)

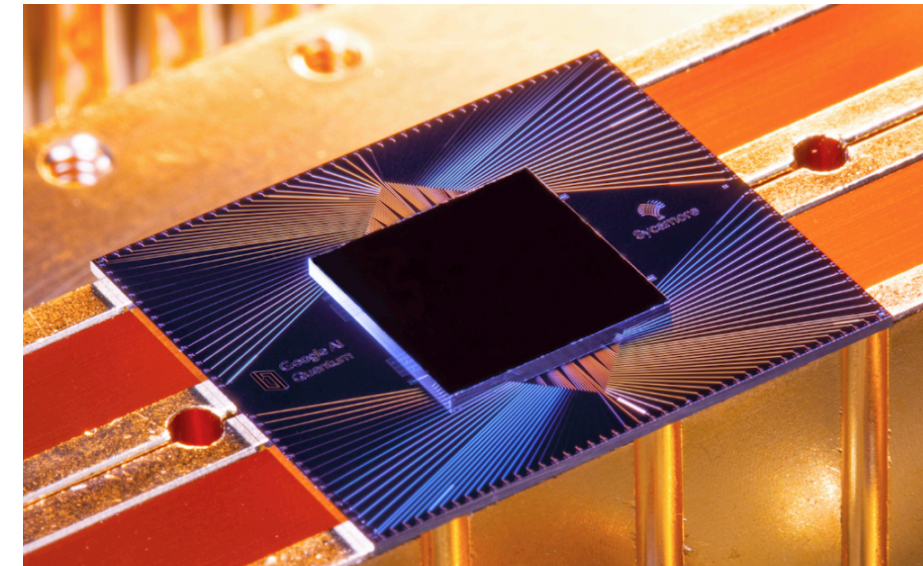


Noisy Intermediate-Scale Quantum Systems



Trapped-ion quantum computer

E.g. Duke/IonQ
Quantinuum

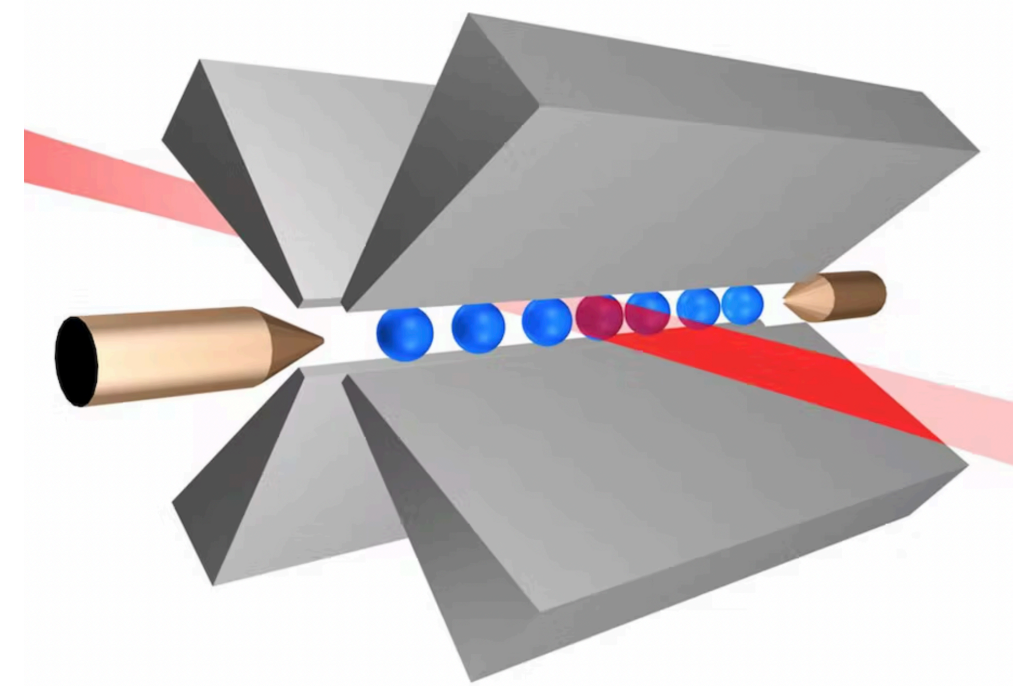


Superconducting qubits

E.g. Google
IBM

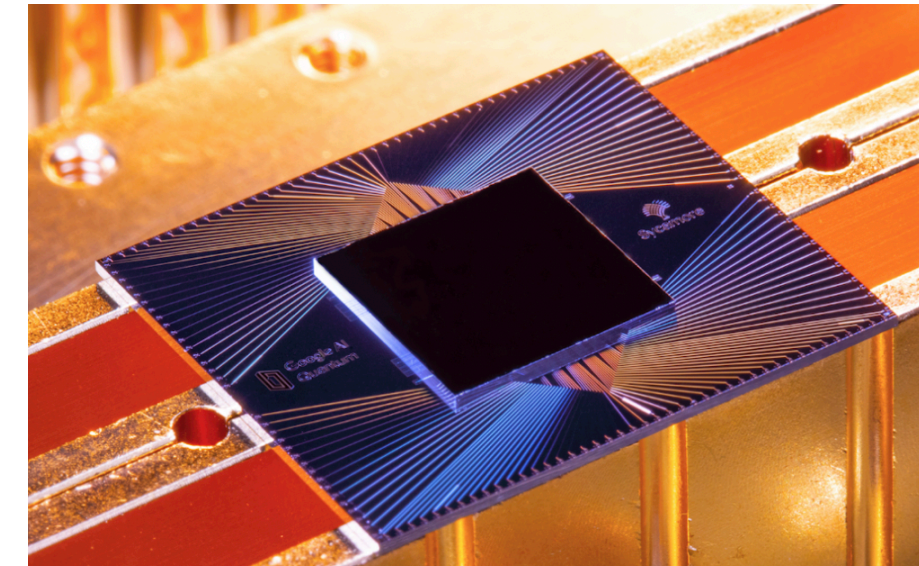
Qubits, unitary gates, mid-circuit measurement available

Noisy Intermediate-Scale Quantum Systems



Trapped-ion quantum computer

E.g. Duke/IonQ
Quantinuum



Superconducting qubits

E.g. Google
IBM

Qubits, unitary gates, mid-circuit measurement available

Can we artificially engineer non-trivial many-body states?

(which may be challenging to find in nature)

(Long-range entangled) Non-trivial states

Example 1 Topological order

No local order parameters

Fractionalized quasiparticles (anyons)

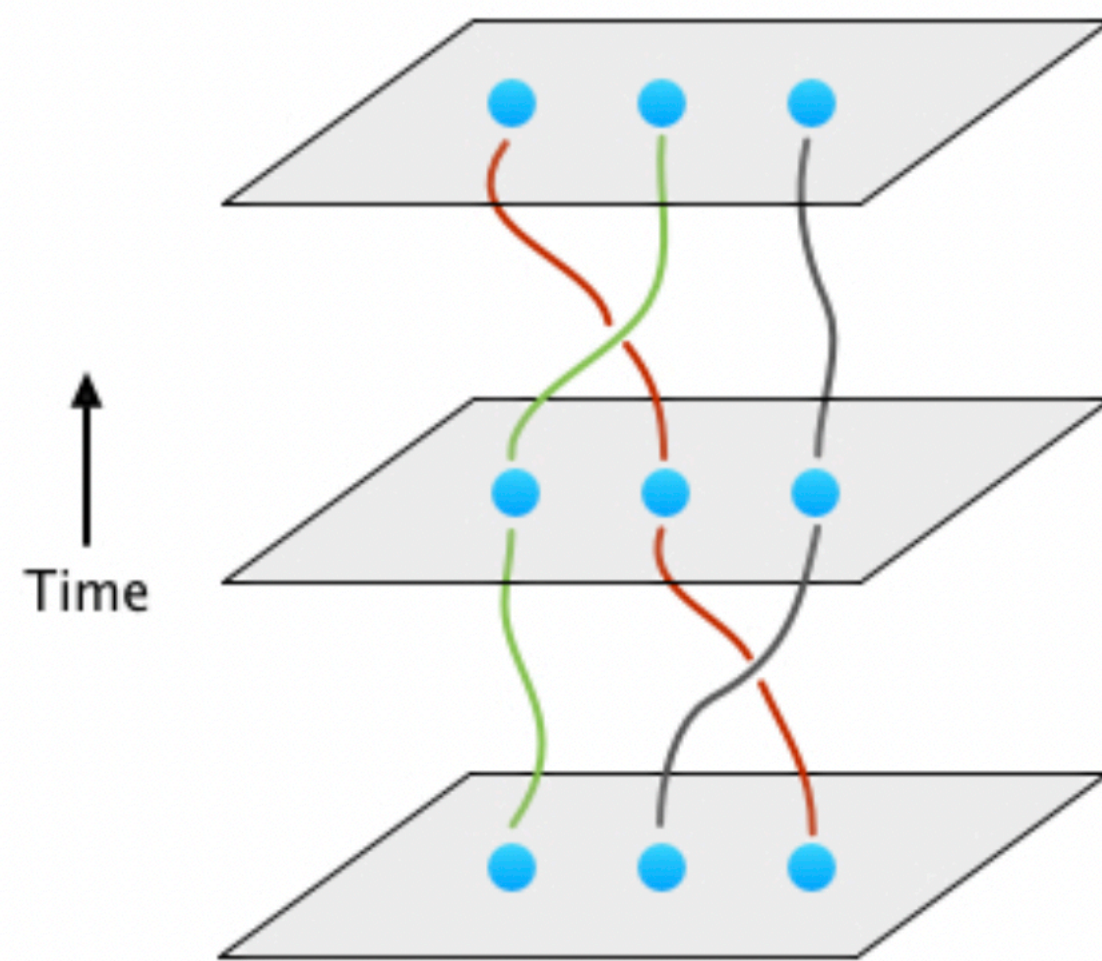


Fig. from Quanta Magazine

(Long-range entangled) Non-trivial states

Example 1 Topological order

No local order parameters

Fractionalized quasiparticles (anyons)

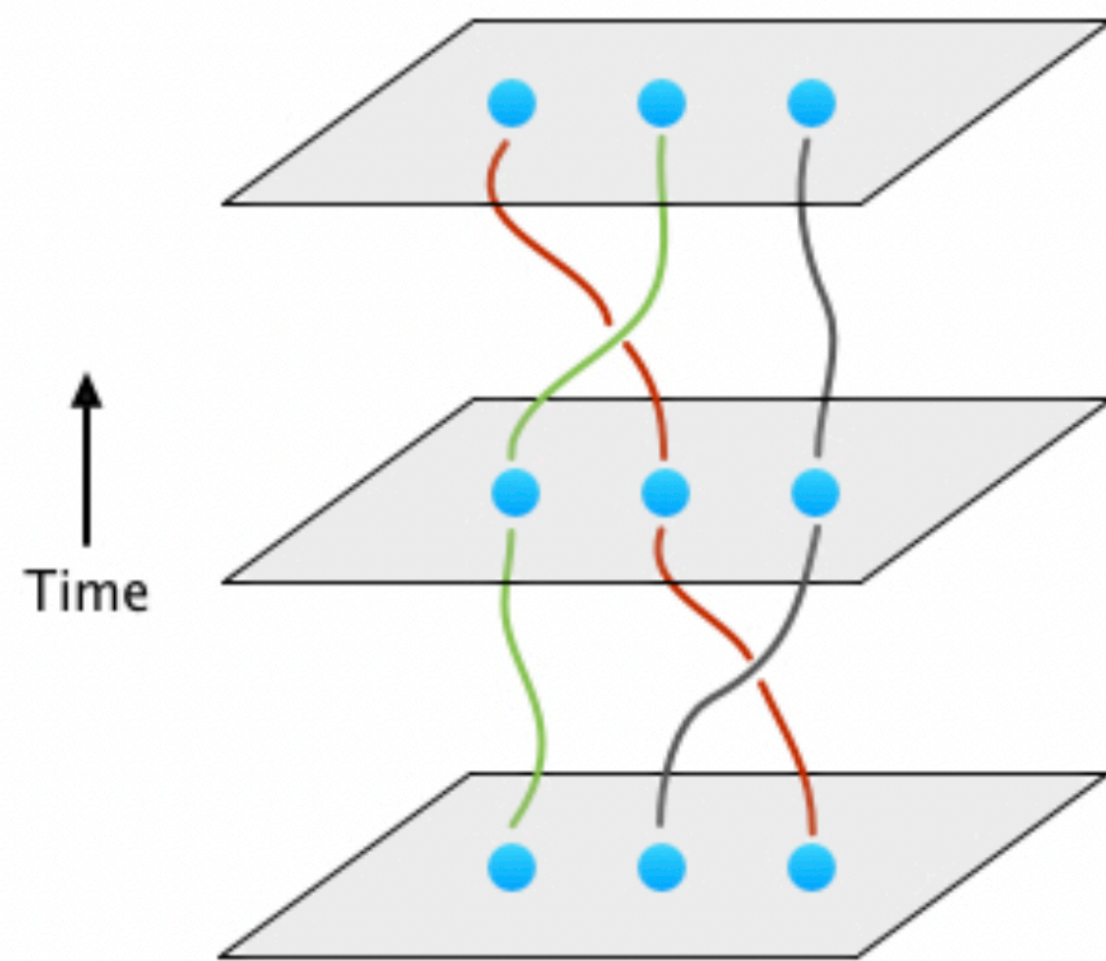


Fig. from Quanta Magazine

Example 2 Quantum critical states

Phase transitions between quantum orders

Quantum correlations at all scales

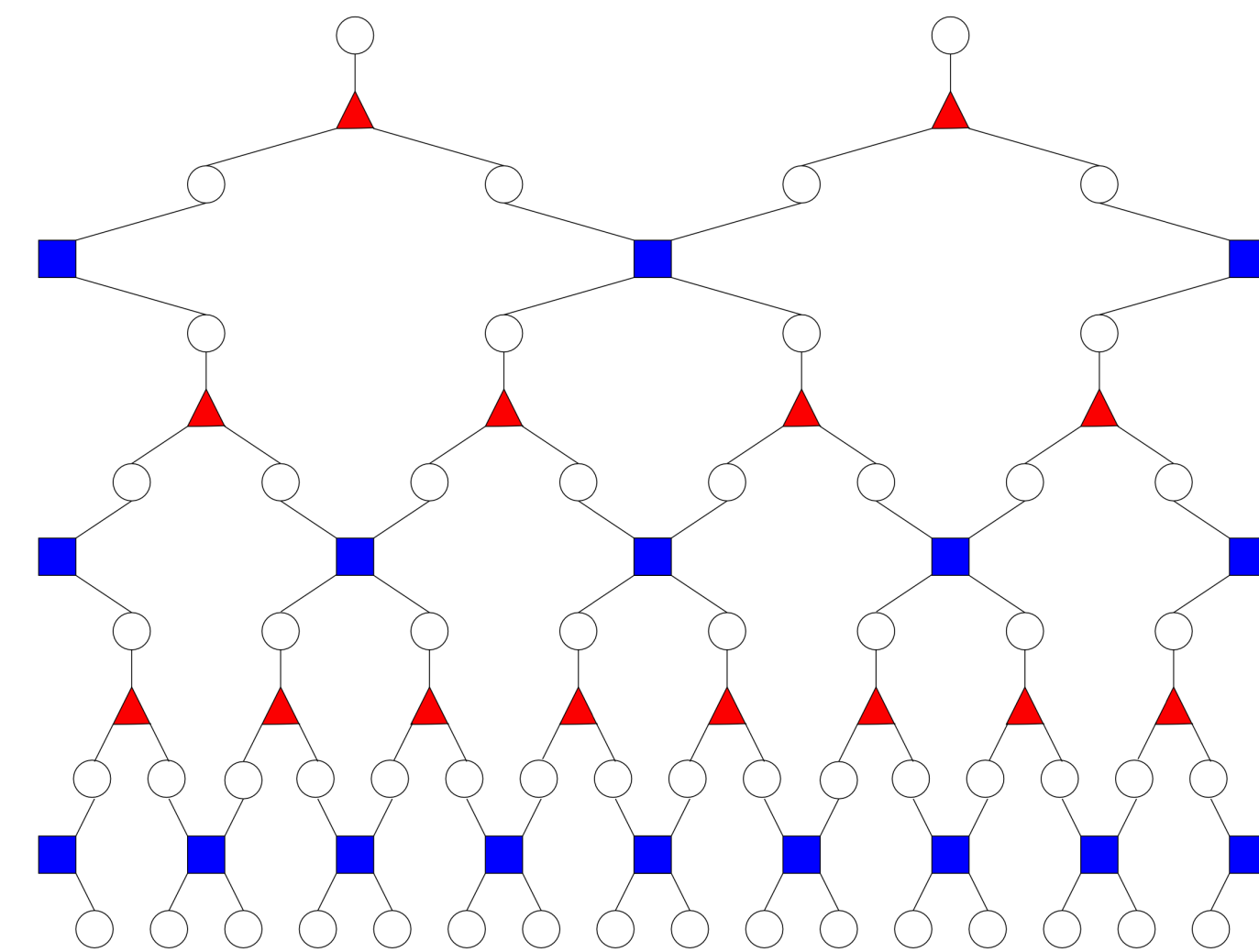


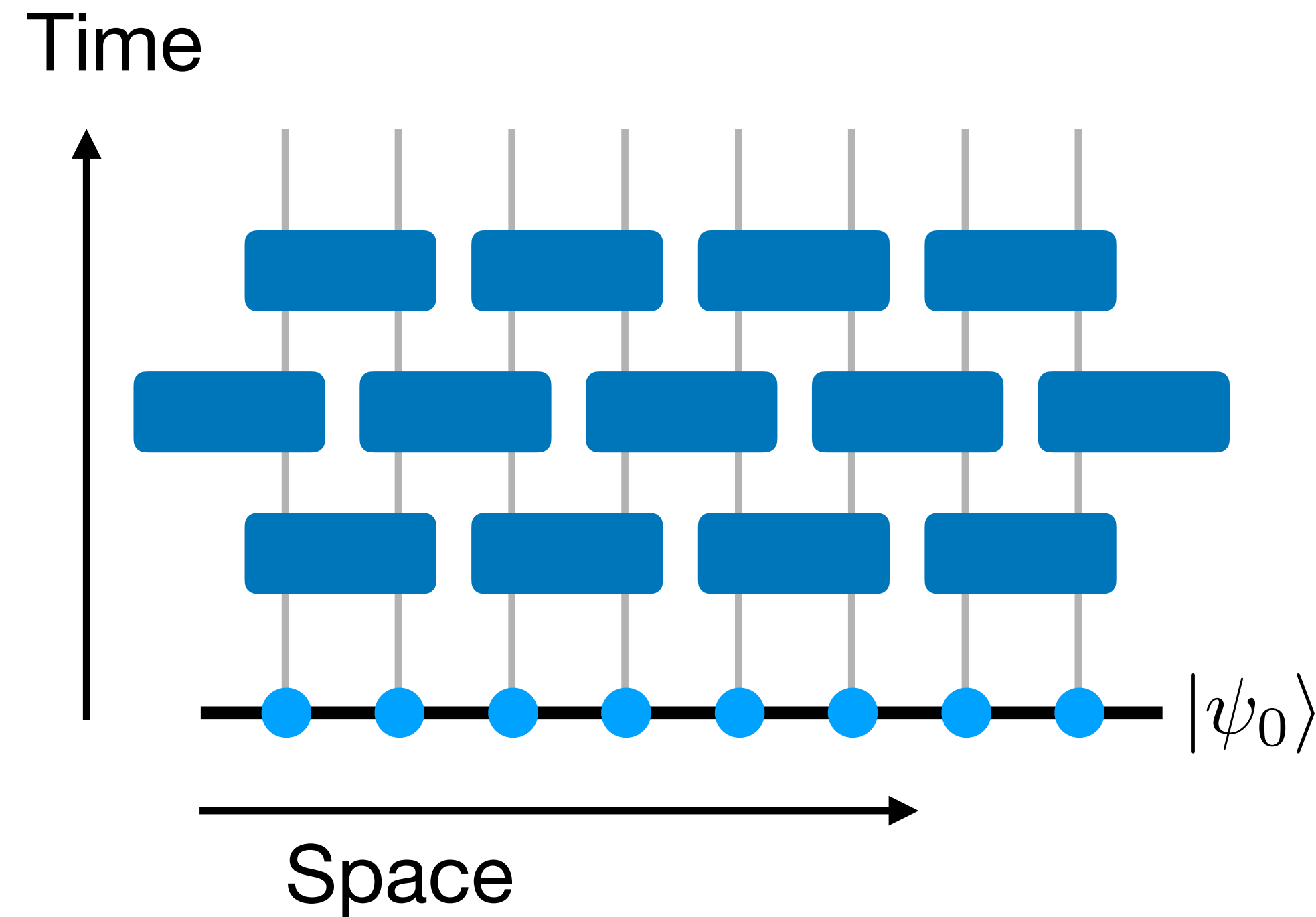
Fig. from Swingle

(Long-range entangled)
Non-trivial states

Example 1
Topological order

Example 2
Quantum critical states

Local unitary circuits



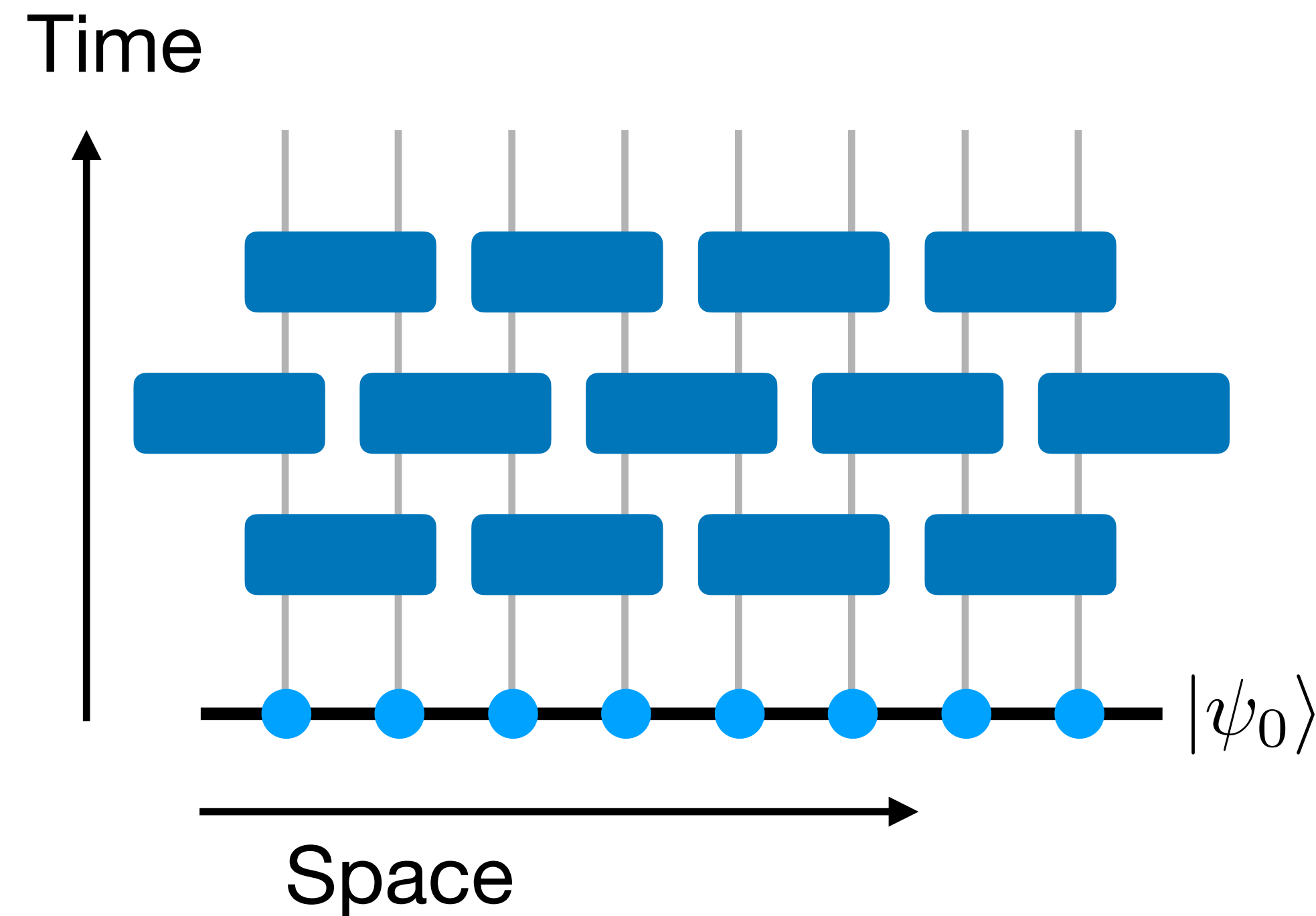
Preparation Time $\gtrsim O(L)$, Lieb-Robinson bound

(Long-range entangled) Non-trivial states

Example 1
Topological order

Example 2
Quantum critical states

Local unitary circuits



Preparation Time $\gtrsim O(L)$, Lieb-Robinson bound

undesirable in near-term quantum computers

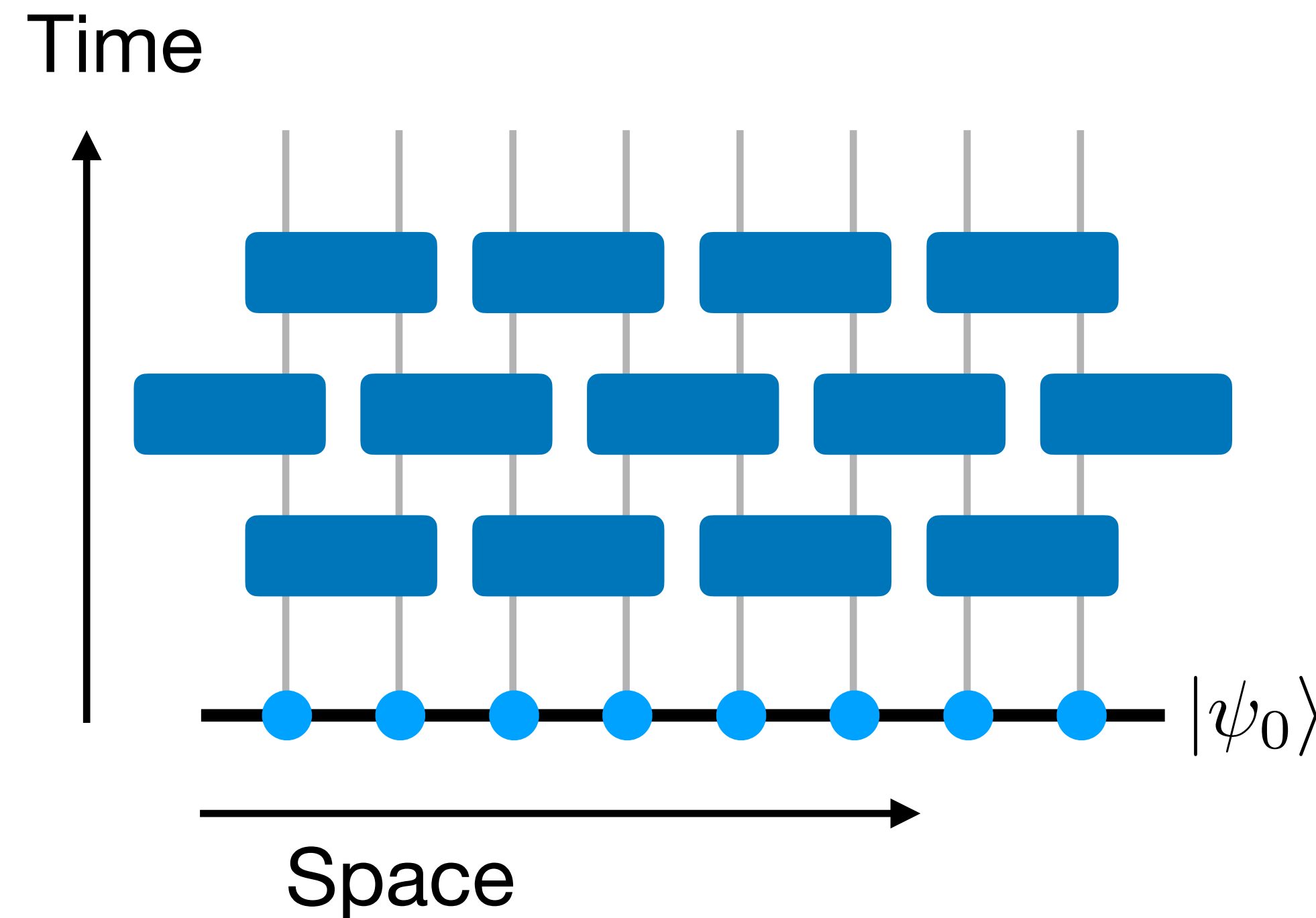
Workaround?

(Long-range entangled) Non-trivial states

Example 1
Topological order

Example 2
Quantum critical states

Local unitary circuits



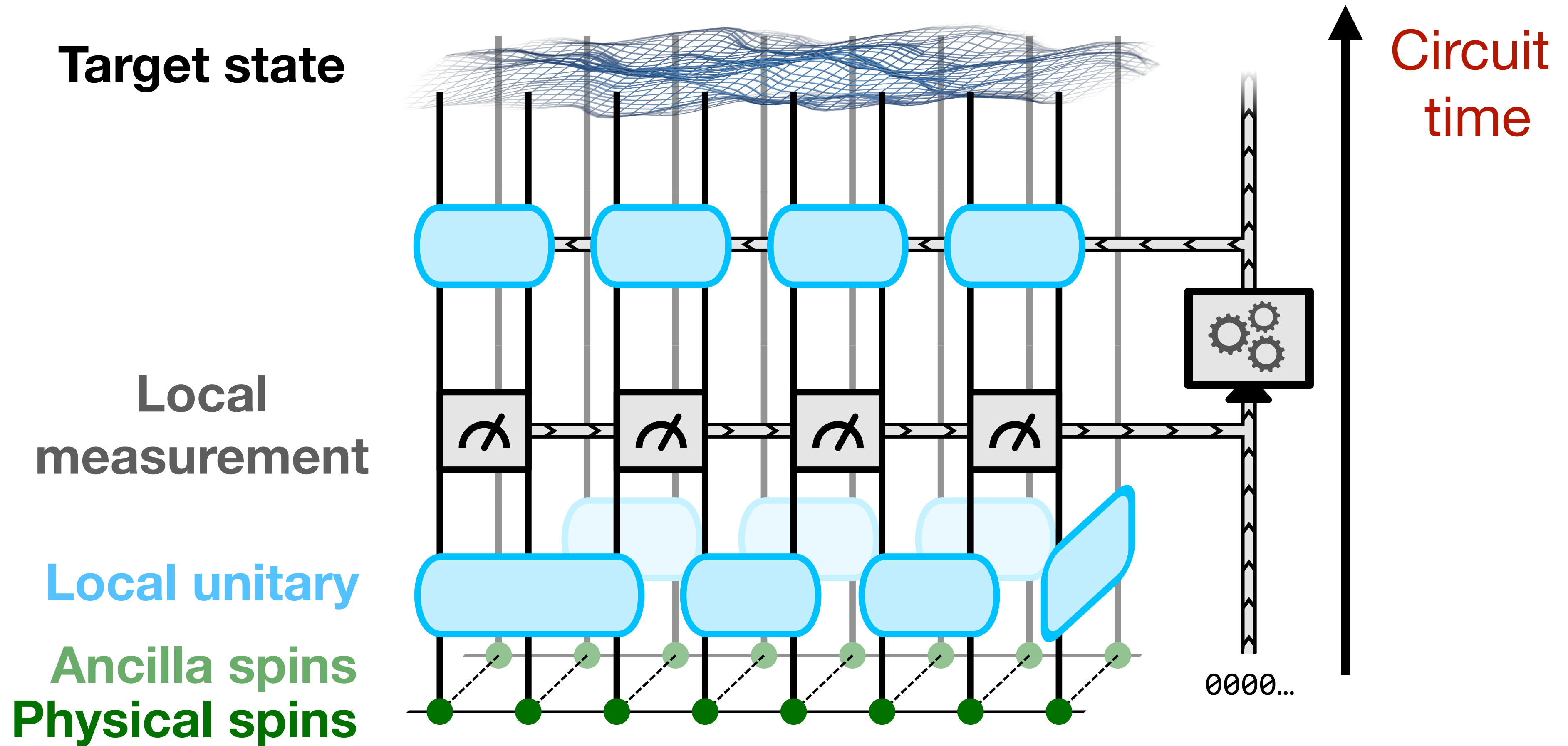
Preparation Time $\gtrsim O(L)$, Lieb-Robinson bound
undesirable in near-term quantum computers

Workaround?

With local **measurement** (non-unitary),
many LRE states are possible in $O(1)$ time !

Adaptive protocols

Local quantum operation + **Non-local classical** communication
(unitary gates & measurement)



The applied unitary gates are **adaptive**
(depend on global measurement outcomes)

Adaptive protocols - (partial) literature

Theory

GHZ state (2001) Briegel, Raussendorf

Toric code (2005) Raussendorf, Bravyi, Harrington

(2021) Piroli, Styliaris, Cirac

Non-abelian (solvable group) T.O. { (2021) Tantivasadakarn, Thorngren, Vishwanath, Verresen
(2022) Bravyi, Kim, Kliesch, Koenig



Adaptive protocols - (partial) literature

Theory

GHZ state (2001) Briegel, Raussendorf

Toric code (2005) Raussendorf, Bravyi, Harrington

(2021) Piroli, Styliaris, Cirac

Non-abelian (solvable group) T.O. { (2021) Tantivasadakarn, Thorngren, Vishwanath, Verresen
(2022) Bravyi, Kim, Kliesch, Koenig

(2022) Lu, Lessa, Kim, Hsieh arXiv: 2206.13527 (PRX Quantum)

Tensor-network, parton construction, entanglement renormalization

Certain (chiral) non-abelian T.O. in $O(1)$ depth

Levin-Wen string nets, CFT gapless (critical) state in $O(\log L)$ depth

Adaptive protocols - (partial) literature

Theory

GHZ state (2001) Briegel, Raussendorf

Toric code (2005) Raussendorf, Bravyi, Harrington

(2021) Piroli, Styliaris, Cirac

Non-abelian (solvable group) T.O.

(2021) Tantivasadakarn, Thorngren, Vishwanath, Verresen

(2022) Bravyi, Kim, Kliesch, Koenig

(2022) Lu, Lessa, Kim, Hsieh arXiv: 2206.13527 (PRX Quantum)

Tensor-network, parton construction, entanglement renormalization

Certain (chiral) non-abelian T.O. in $O(1)$ depth

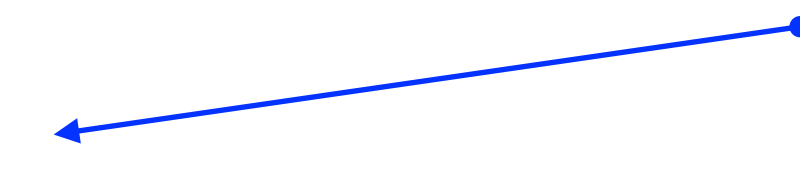
Levin-Wen string nets, CFT gapless (critical) state in $O(\log L)$ depth

Experiment

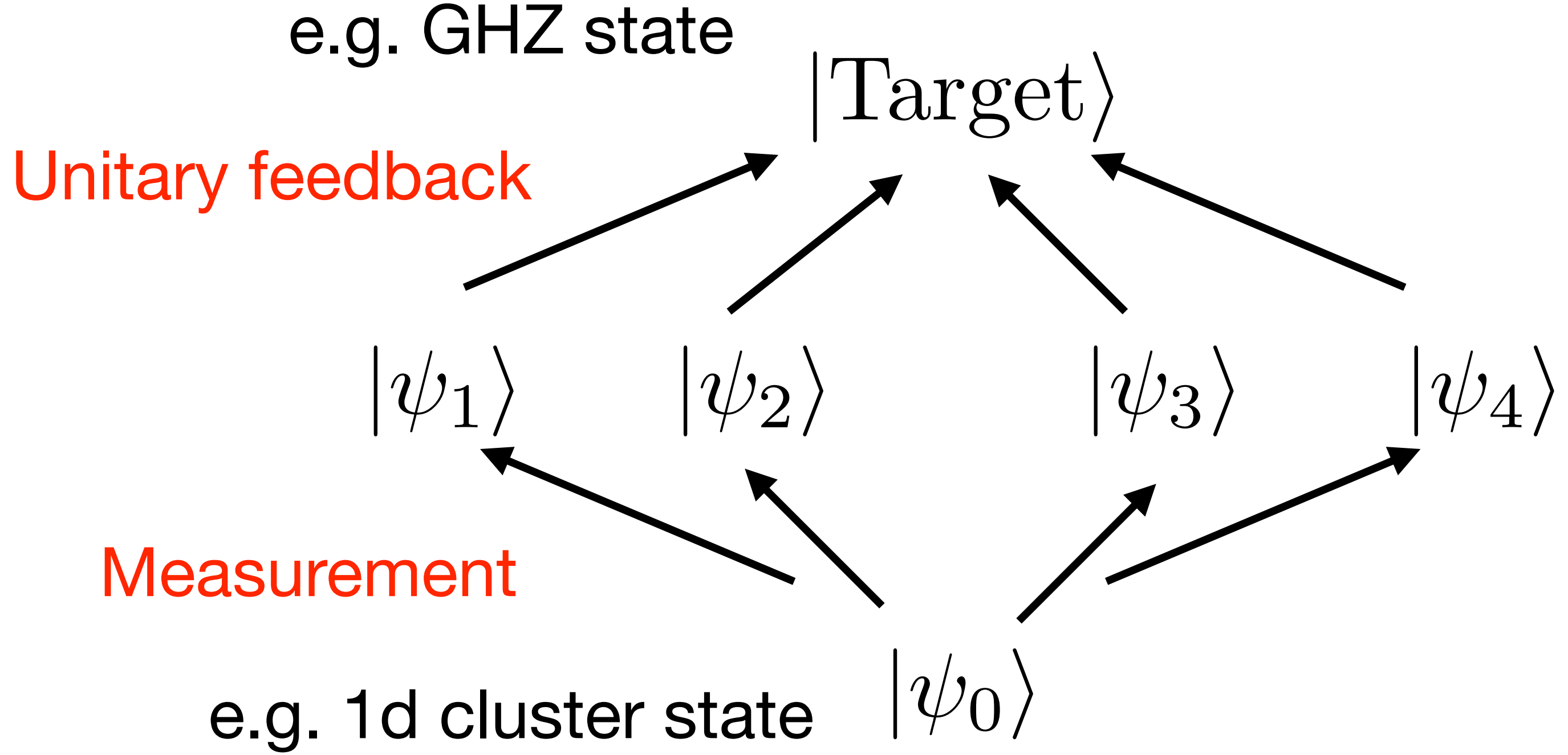
Toric code (2023) Quantinuum, **Perimeter**, UC Davis

(2023) Quantinuum, Harvard, Caltech

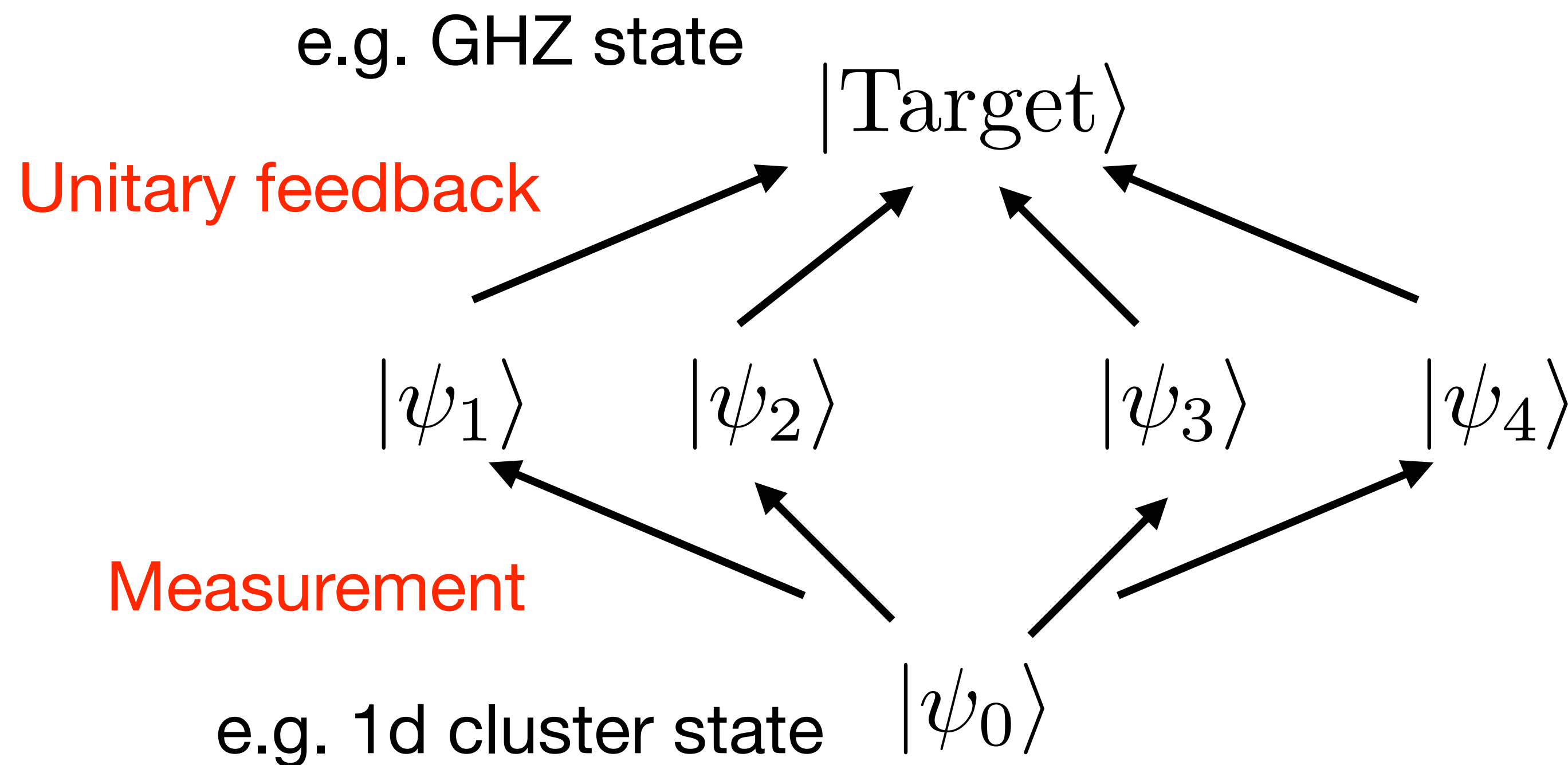
Non-abelian T.O.



Stability of adaptive protocols?



Stability of adaptive protocols?



Fine-tuning required

When perturbing initial state,
pure-state trajectories
may not converge

➡ Realizing a mixed state ρ

Long-range entangled?

This talk

arXiv: 2303.15507 (PRX Quantum), **Lu**, Zhang, Vijay, Hsieh

Simple **pure** states

e.g. short-range entangled

$O(1)$ -depth
channel

Non-trivial (LRE) **mixed** states

Quantum order/criticality
coexisting with **classical fluctuations**

This talk

arXiv: 2303.15507 (PRX Quantum), **Lu**, Zhang, Vijay, Hsieh

Simple **pure** states

e.g. short-range entangled

**O(1)-depth
channel**

Non-trivial (LRE) **mixed** states

Quantum order/criticality
coexisting with **classical fluctuations**

In contrast

$$\rho \sim e^{-\beta H}$$

Typically short-range entangled

e.g. 2d toric code, Hastings (2011)

This talk

arXiv: 2303.15507 (PRX Quantum), **Lu**, Zhang, Vijay, Hsieh

Simple **pure** states

e.g. short-range entangled

**O(1)-depth
channel**

Non-trivial (LRE) **mixed** states

Quantum order/criticality
coexisting with **classical fluctuations**

In contrast

$$\rho \sim e^{-\beta H}$$

Typically short-range entangled

e.g. 2d toric code, Hastings (2011)

Route to non-trivial mixed-state
phases of matter

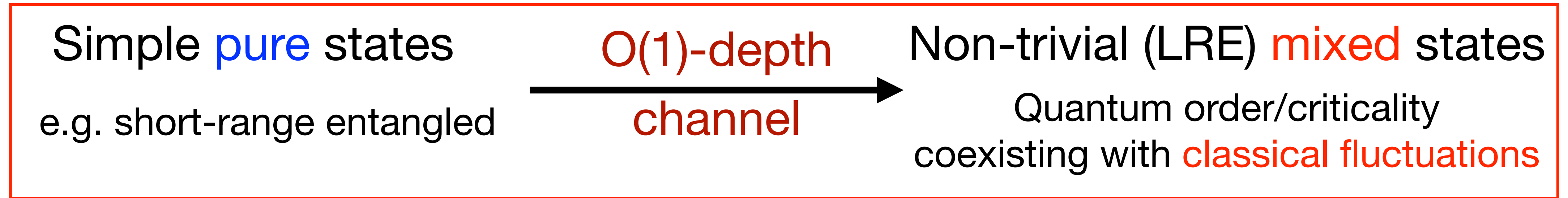
Recent interest:

(pure) many-body states (e.g. SPT, topological order)
under decoherence

de Groot, Turzillo, Schuch (2022); Lee, You, Xu (2022);
Zhang, Qi, Bi (2022); Ma, Wang (2022); Fan, Bao, Altman, Vishwanath (2023) ...

This talk

arXiv: 2303.15507 (PRX Quantum), **Lu**, Zhang, Vijay, Hsieh



Inspired by Briegel, Raussendorf (2001);
Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021)

$Z_2 \times Z_2$ SPT order
(symmetry-protected topological)



Z_2 long-range
(topological) order

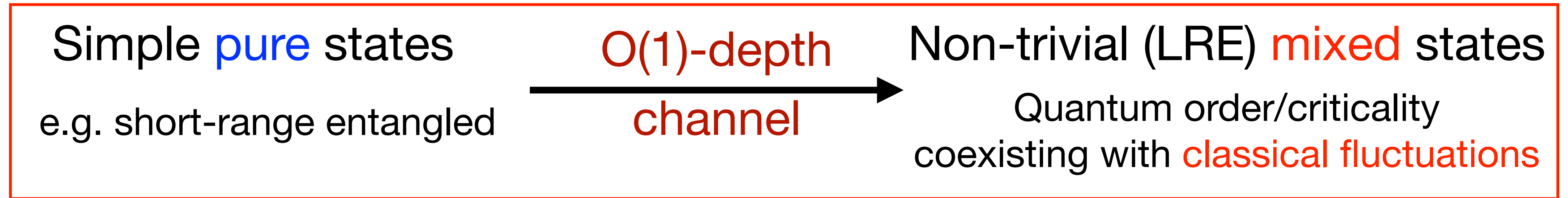
Gapped (pure) state
(Chern insulators)



Quantum critical mixed states

This talk

arXiv: 2303.15507 (PRX Quantum), Lu, Zhang, Vijay, Hsieh



Inspired by Briegel, Raussendorf (2001);
Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021)

$Z_2 \times Z_2$ SPT order
(symmetry-protected topological)



Z_2 long-range
(topological) order

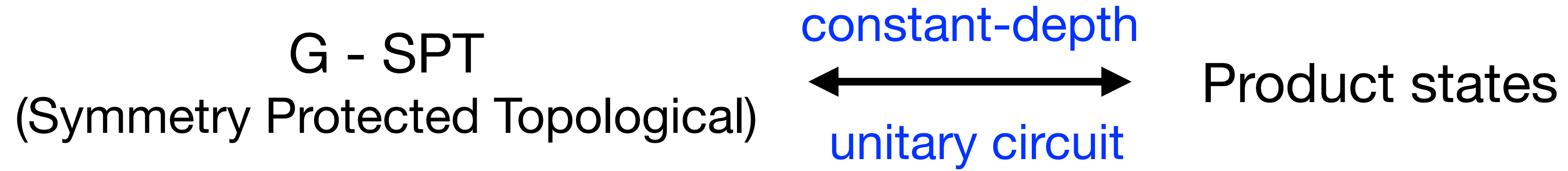
Gapped (pure) state
(Chern insulators)



Quantum critical mixed states

Emergence of order/criticality is a **universal** property of the input phase
hence demonstrating robustness

Application with SPT



Only when the gates breaks G symmetry

Application with SPT

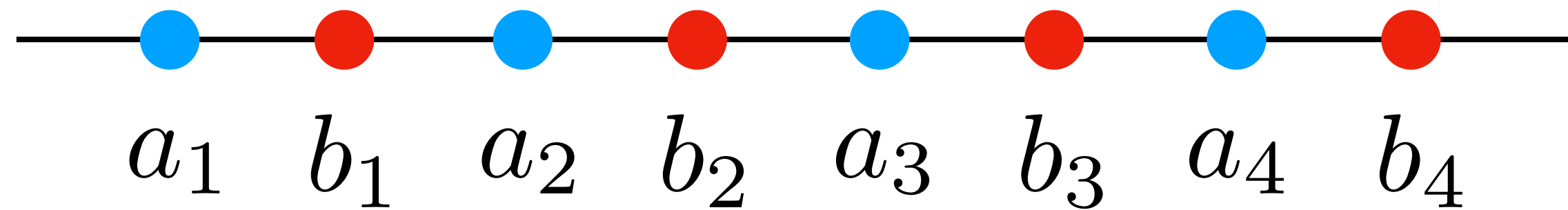
G - SPT
(Symmetry Protected Topological)

constant-depth
 \longleftrightarrow
unitary circuit

Product states

Only when the gates breaks G symmetry

$G = Z_2 \times Z_2$
Cluster-state SPT



$$\begin{aligned}
 & - \sum_i Z_{a,i} X_{b,i} Z_{a,i+1} - \sum_i Z_{b,i-1} X_{a,i} Z_{b,i} \quad \text{Symmetry} \quad \prod_i X_{a,i} \quad \prod_i X_{b,i}
 \end{aligned}$$

Application with SPT

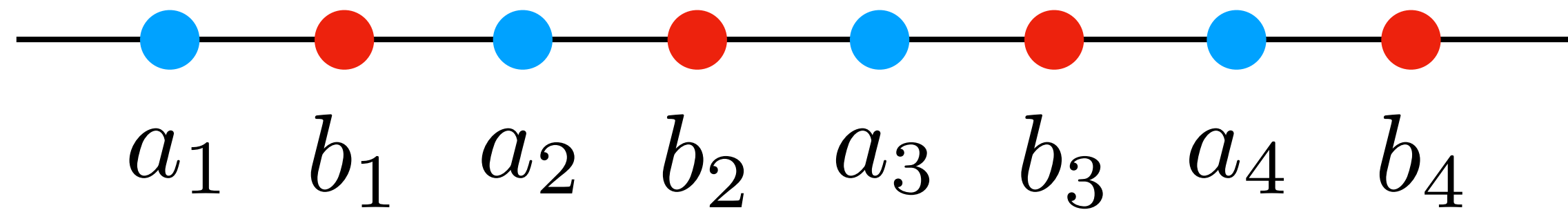
G - SPT
(Symmetry Protected Topological)

constant-depth
←————→
unitary circuit

Product states

Only when the gates breaks G symmetry

$G = Z_2 \times Z_2$
Cluster-state SPT



$$-\sum_i Z_{a,i} X_{b,i} Z_{a,i+1} - \sum_i Z_{b,i-1} X_{a,i} Z_{b,i}$$

Symmetry $\prod_i X_{a,i} \prod_i X_{b,i}$

Long-range string order

$$\langle Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \rangle = 1$$

See e.g. Pollmann, Turner (2012)

Application with SPT

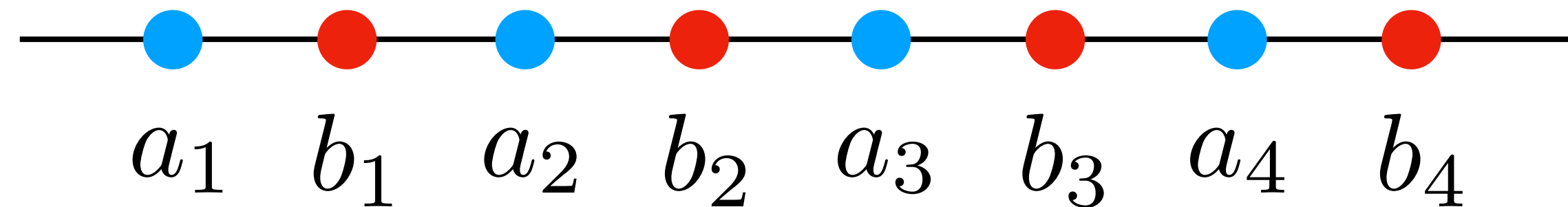
G - SPT
(Symmetry Protected Topological)

constant-depth
←————→
unitary circuit

Product states

Only when the gates breaks G symmetry

$G = Z_2 \times Z_2$
Cluster-state SPT



$$-\sum_i Z_{a,i} X_{b,i} Z_{a,i+1} - \sum_i Z_{b,i-1} X_{a,i} Z_{b,i}$$

Symmetry $\prod_i X_{a,i} \prod_i X_{b,i}$

$-g \sum_i O_i$ Long-range string order $\langle Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \rangle = c = O(1)$

See e.g. Pollmann, Turner (2012)

Application with SPT

Long-range string order
SPT $|\psi_0\rangle$

$$\langle Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \rangle = c = O(1) > 0$$



(GHZ-like) mixed state

ρ_B

$$\langle Z_{b,i} Z_{b,j} \rangle = c = O(1) > 0, \quad \langle \prod_i X_{b,i} \rangle = 1$$

e.g. GHZ state $|000\dots\rangle + |111\dots\rangle$

$$c = 1$$

Application with SPT

Long-range string order
SPT $|\psi_0\rangle$

$$\langle Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \rangle = c = O(1) > 0$$



(GHZ-like) mixed state

$$\rho_B \quad \langle Z_{b,i} Z_{b,j} \rangle = c = O(1) > 0, \quad \langle \prod_i X_{b,i} \rangle = 1$$

Precise meaning of non-trivial (LRE) mixed state Hastings, PRL (2011)

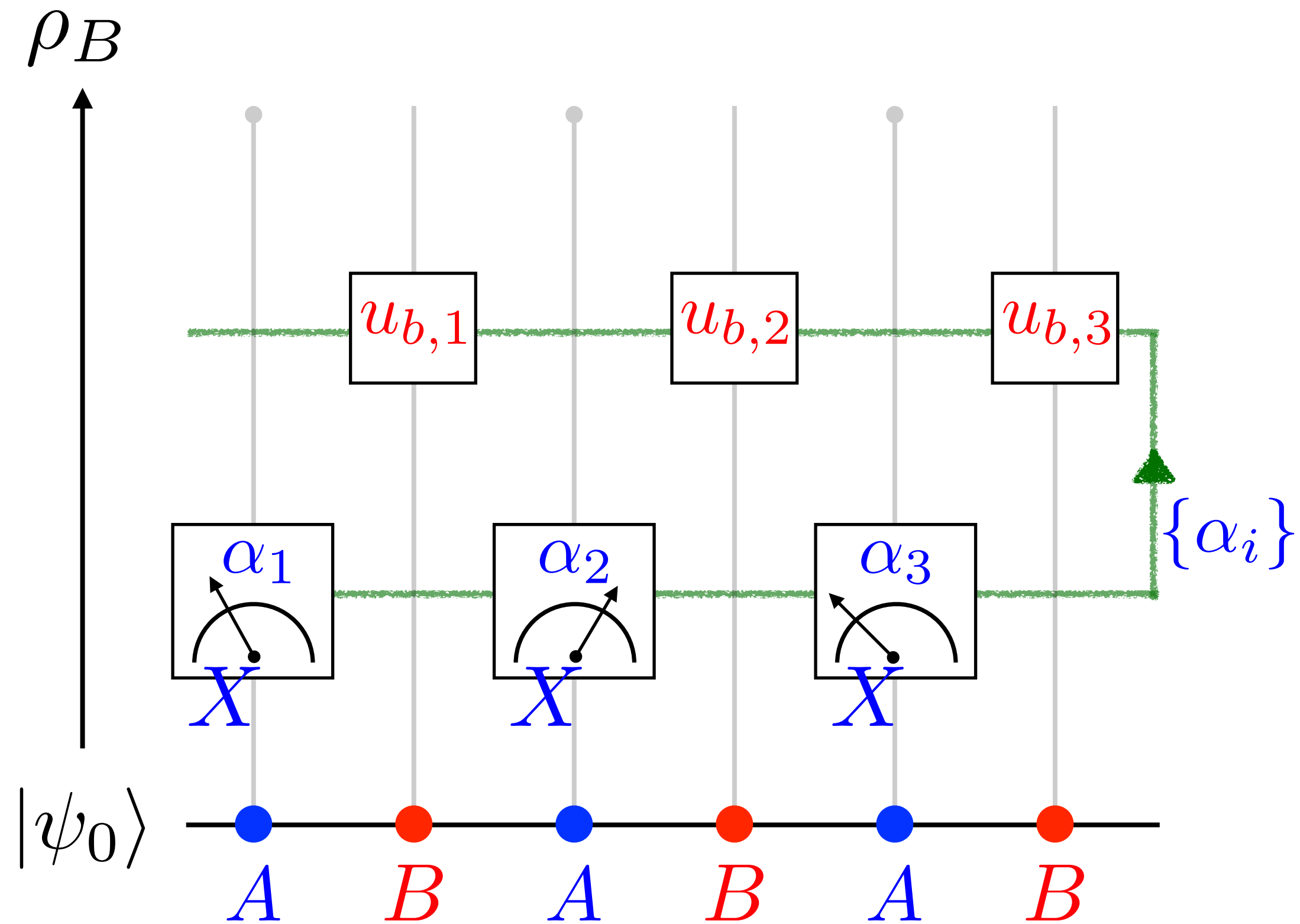
$$\rho_B \neq \sum_n p_n |\text{SRE}_n\rangle \langle \text{SRE}_n| \quad |\text{SRE}_n\rangle \text{ is connected to } \uparrow\uparrow\uparrow\uparrow \dots$$

using local unitary in constant time

Depth-2 protocol

Mixed-state ensemble

$$\left\{ p_\alpha, |\psi_\alpha\rangle = \frac{U_\alpha P_\alpha |\psi_0\rangle}{\sqrt{\langle \psi_0 | P_\alpha | \psi_0 \rangle}} \right\}$$



Depth-2 protocol

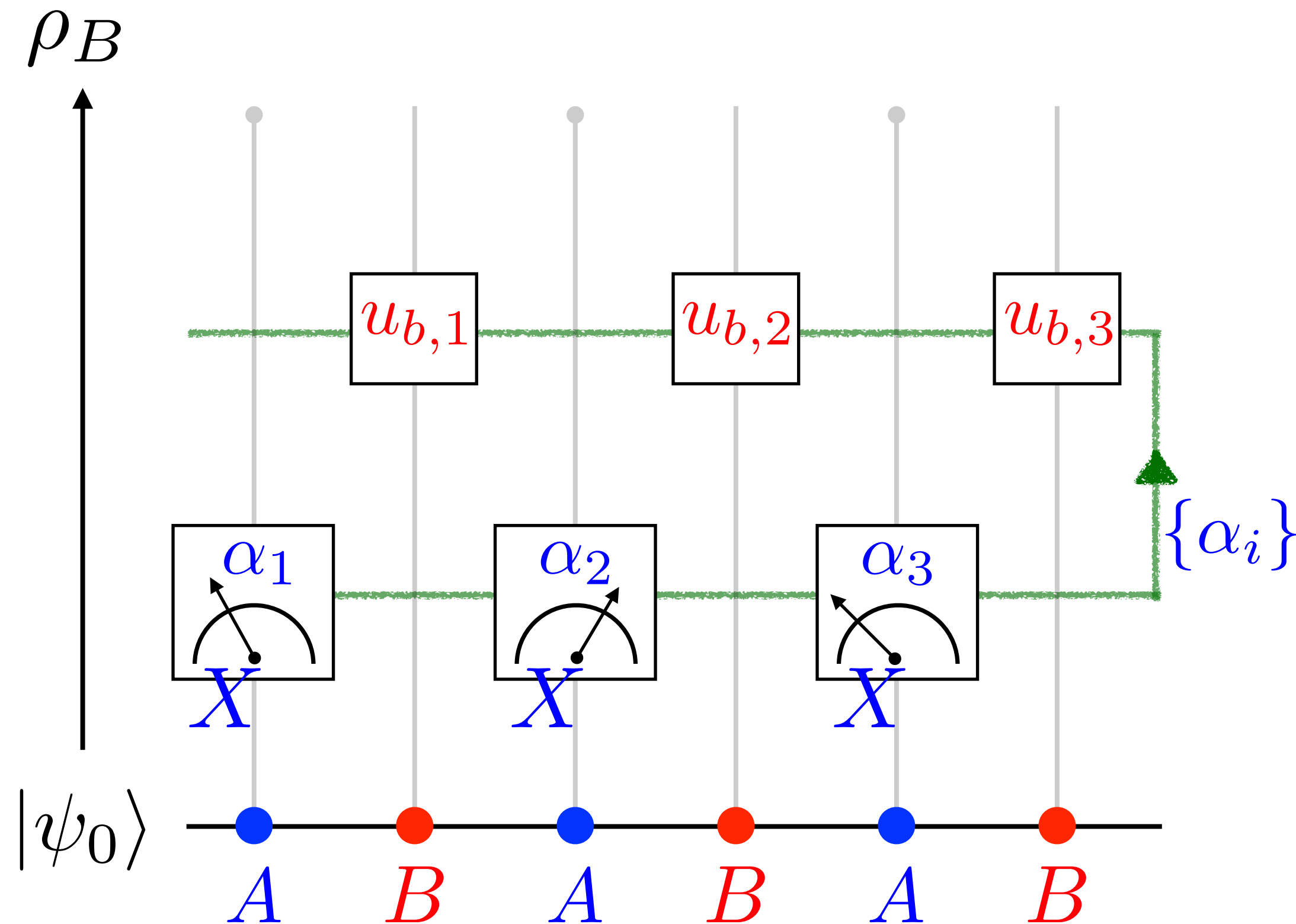
Mixed-state ensemble

$$\left\{ p_\alpha, |\psi_\alpha\rangle = \frac{U_\alpha P_\alpha |\psi_0\rangle}{\sqrt{\langle \psi_0 | P_\alpha | \psi_0 \rangle}} \right\}$$

$$\rho_B = \text{tr}_A (|\psi\rangle\langle\psi|)$$

Purification $|\psi\rangle = \sum_\alpha U_\alpha P_\alpha |\psi_0\rangle$

A controlled unitary U
cannot be realized using
finite-depth unitary circuits



Depth-2 protocol

Mixed-state ensemble

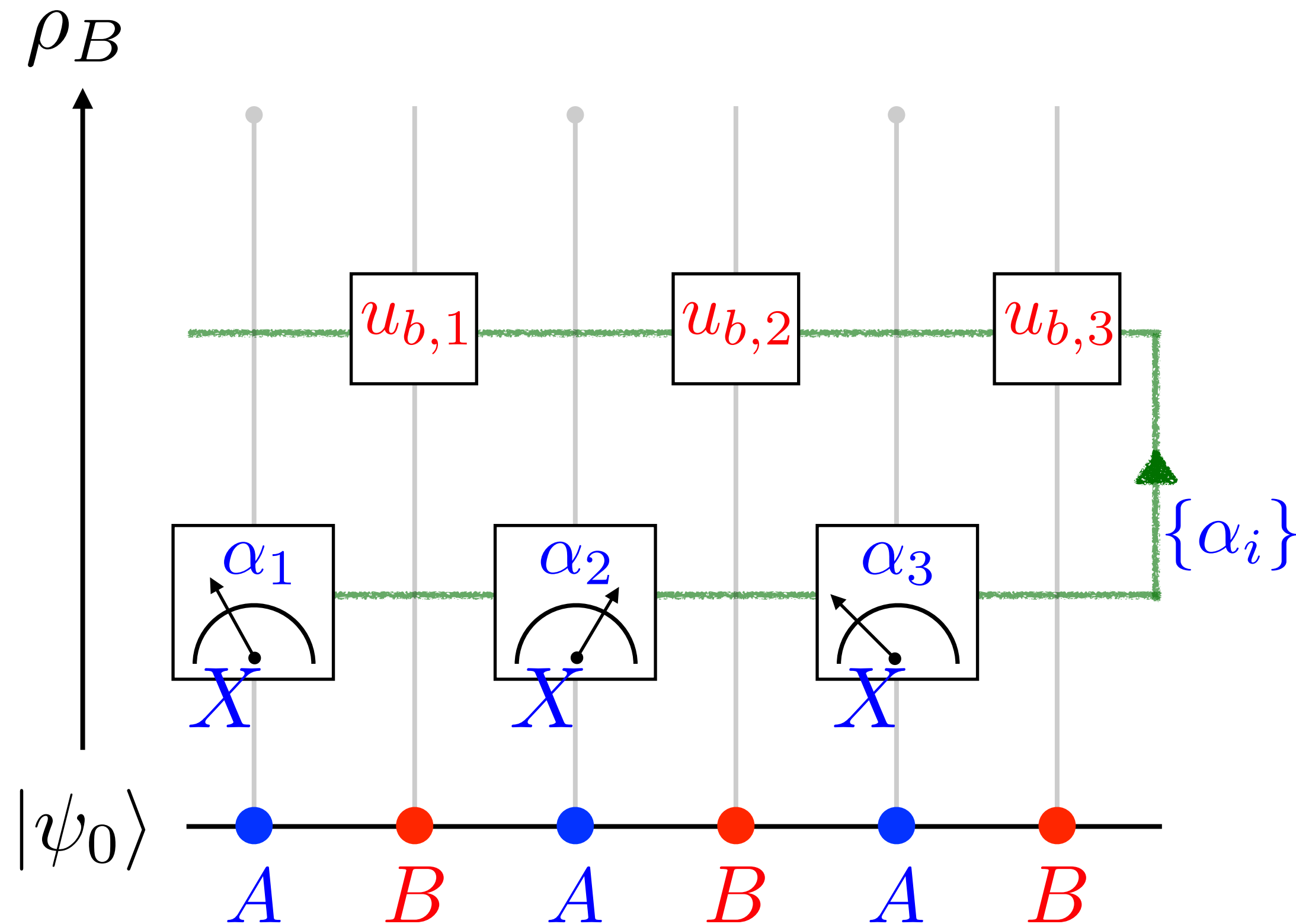
$$\left\{ p_\alpha, |\psi_\alpha\rangle = \frac{U_\alpha P_\alpha |\psi_0\rangle}{\sqrt{\langle \psi_0 | P_\alpha | \psi_0 \rangle}} \right\}$$

$$\rho_B = \text{tr}_A (|\psi\rangle\langle\psi|)$$

Purification $|\psi\rangle = \sum_\alpha U_\alpha P_\alpha |\psi_0\rangle$

A controlled unitary U

cannot be realized using finite-depth unitary circuits



Feedback $U_\alpha = \prod_i X_{b,i}^{\frac{1 - \prod_{j=1,2,\dots}^i \alpha_j}{2}}$

$$U_\alpha^\dagger Z_{b,i} U_\alpha = (\alpha_1 \alpha_2 \cdots \alpha_i) Z_{b,i}$$

Depth-2 protocol

Mixed-state ensemble

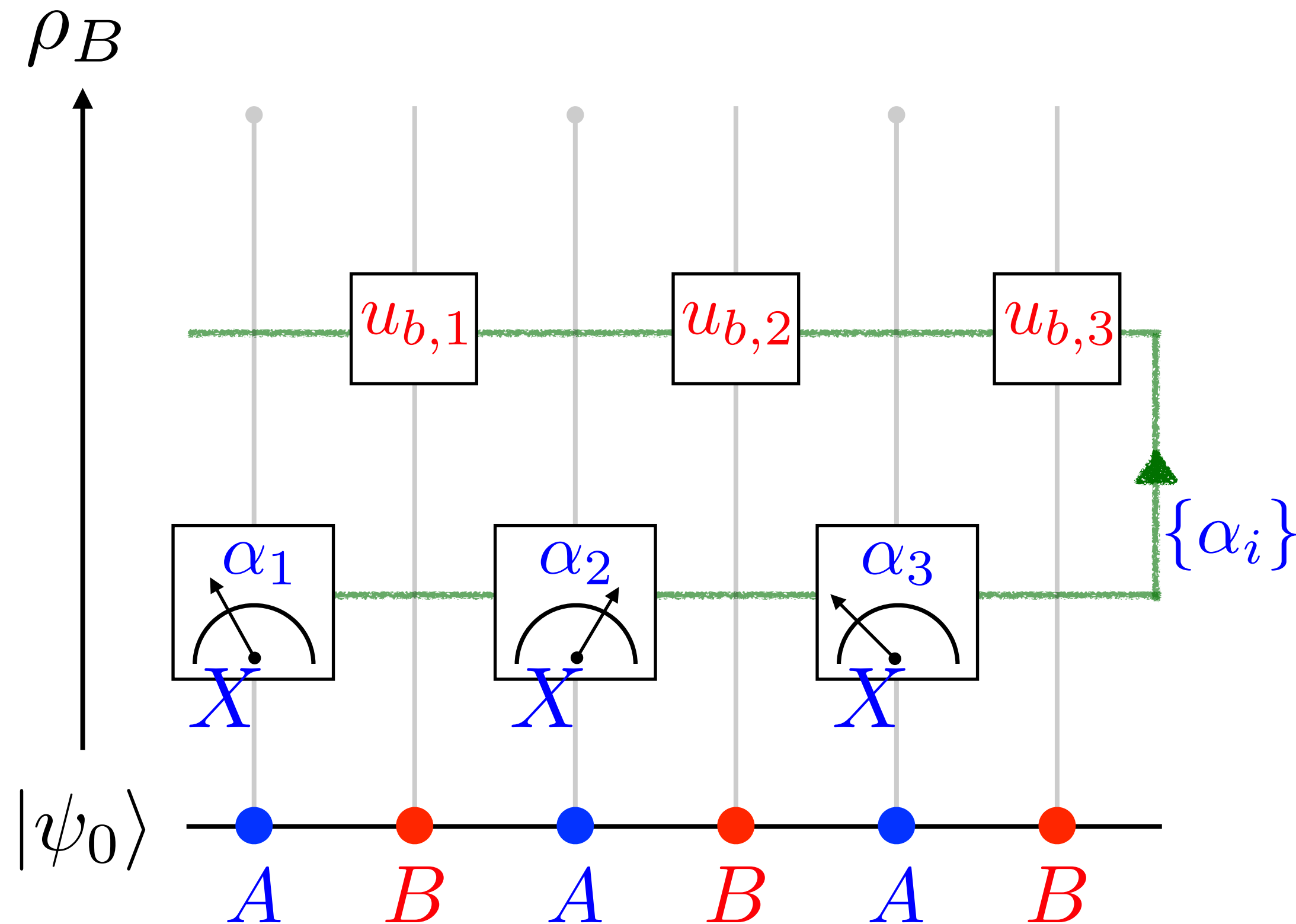
$$\left\{ p_\alpha, |\psi_\alpha\rangle = \frac{U_\alpha P_\alpha |\psi_0\rangle}{\sqrt{\langle \psi_0 | P_\alpha | \psi_0 \rangle}} \right\}$$

$$\rho_B = \text{tr}_A (|\psi\rangle\langle\psi|)$$

Purification $|\psi\rangle = \sum_\alpha U_\alpha P_\alpha |\psi_0\rangle$

A controlled unitary U

cannot be realized using finite-depth unitary circuits

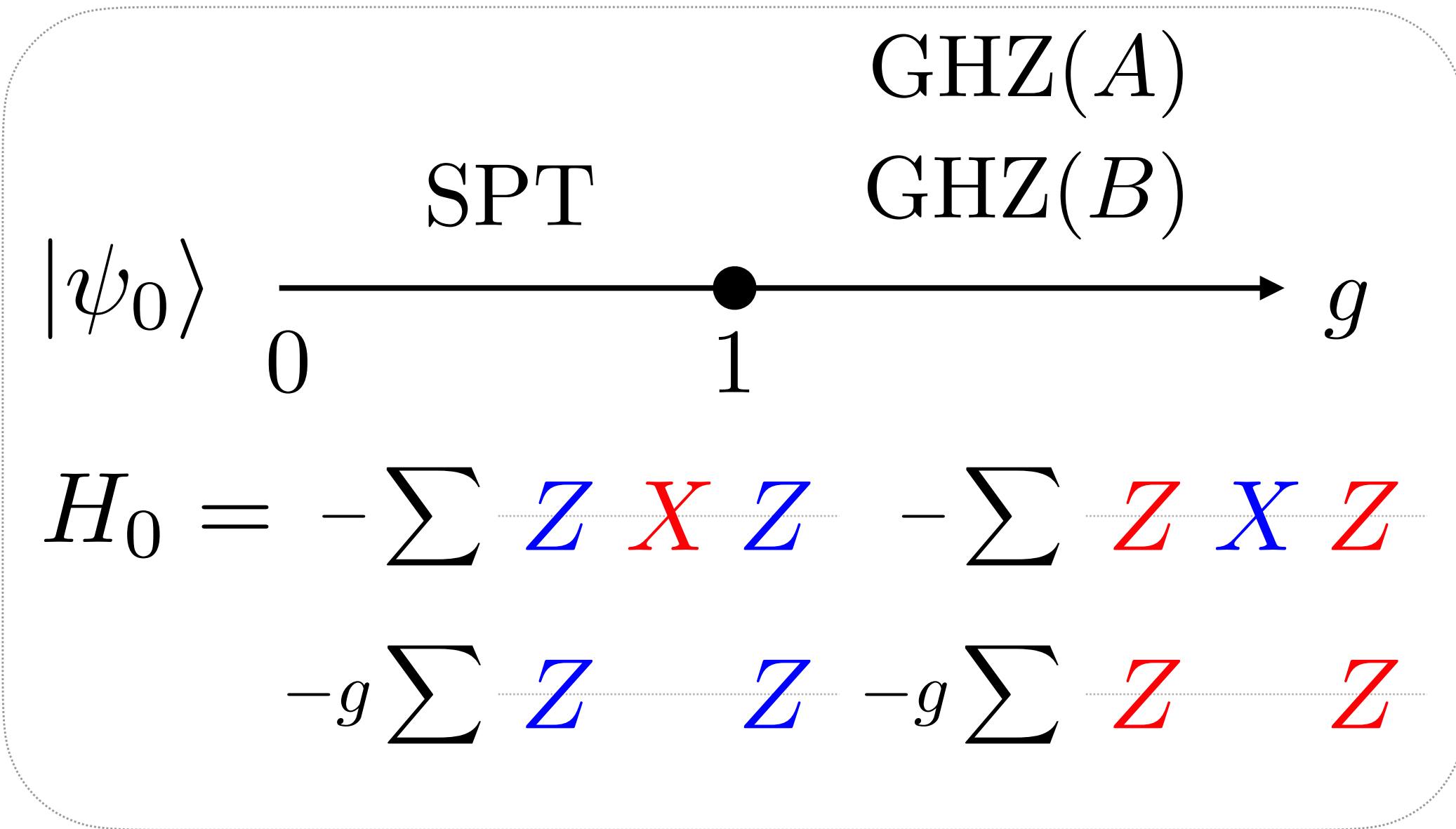



Feedback
$$U_\alpha = \prod_i X_{b,i}^{\frac{1 - \prod_{j=1,2,\dots}^i \alpha_j}{2}}$$

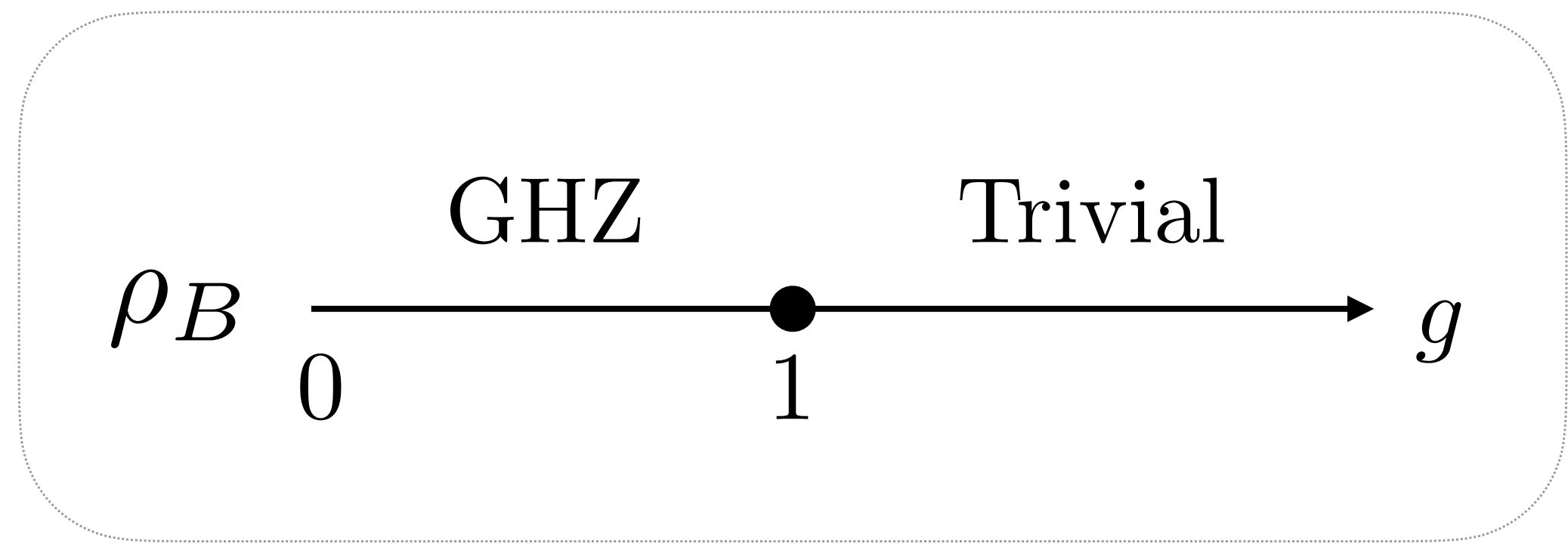
$$U_\alpha^\dagger Z_{b,i} U_\alpha = (\alpha_1 \alpha_2 \cdots \alpha_i) Z_{b,i}$$

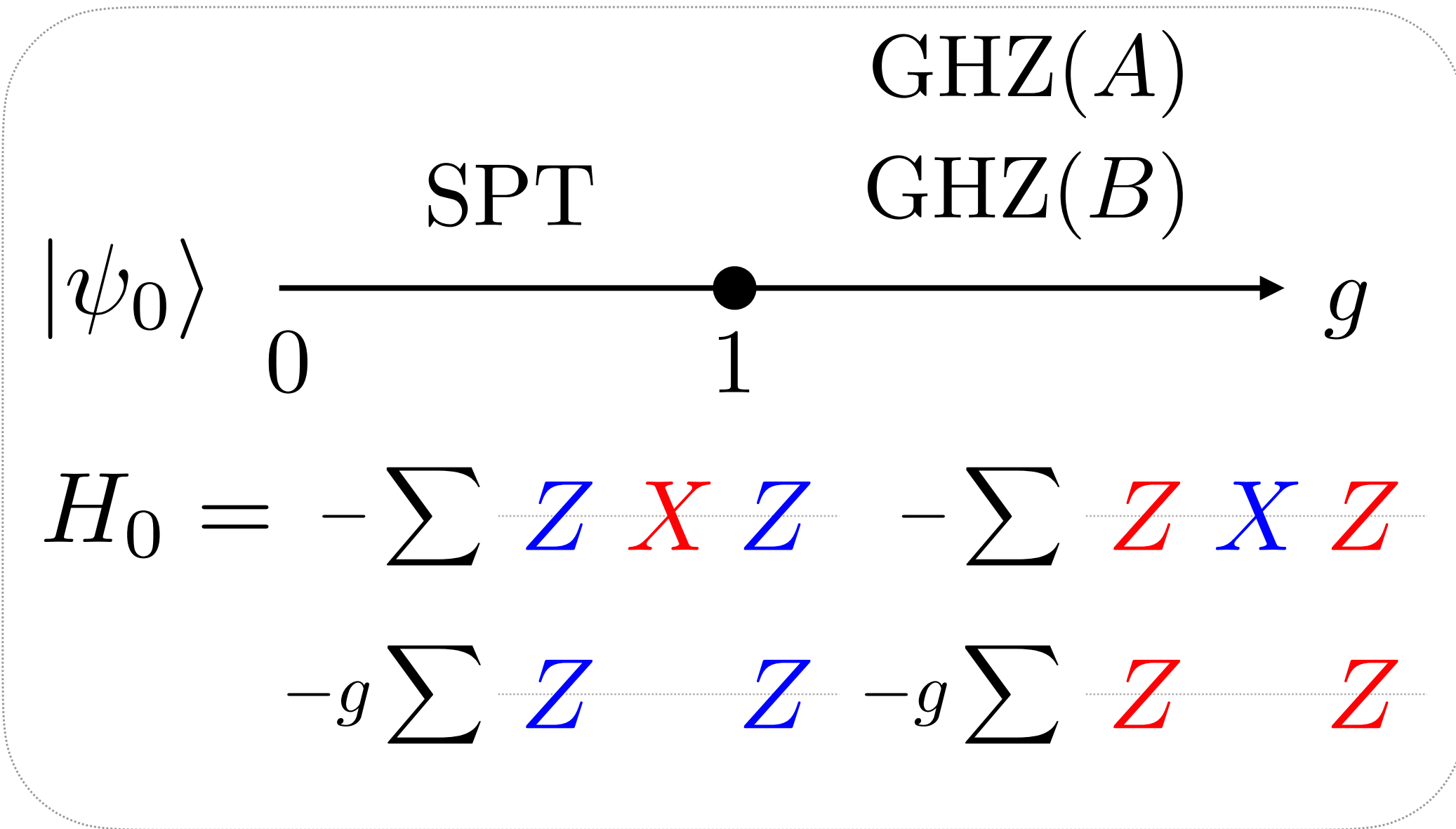
$$U^\dagger \left[Z_{b,i} \left(\prod_{k=i+1}^j X_{a,k} \right) Z_{b,j} \right] U = Z_{b,i} Z_{b,j}$$


Hidden string order \rightarrow long-range order

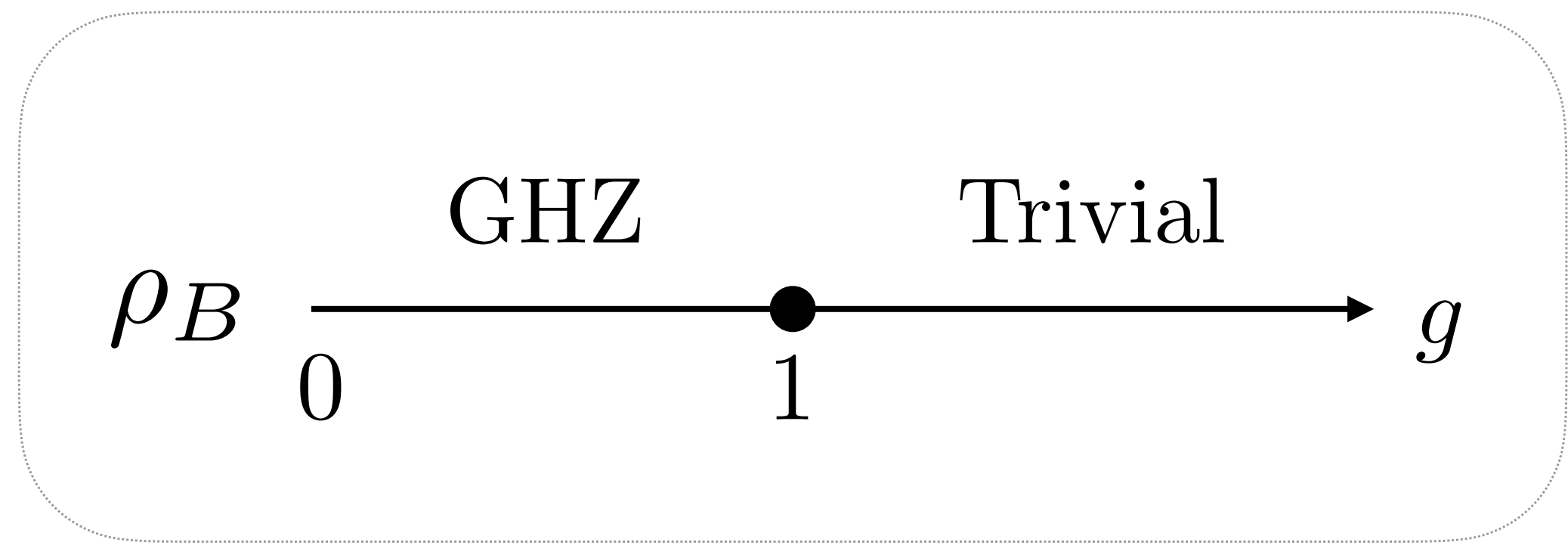


Measurement

 + Feedback



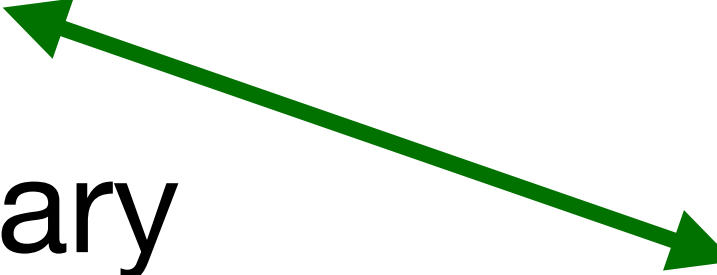
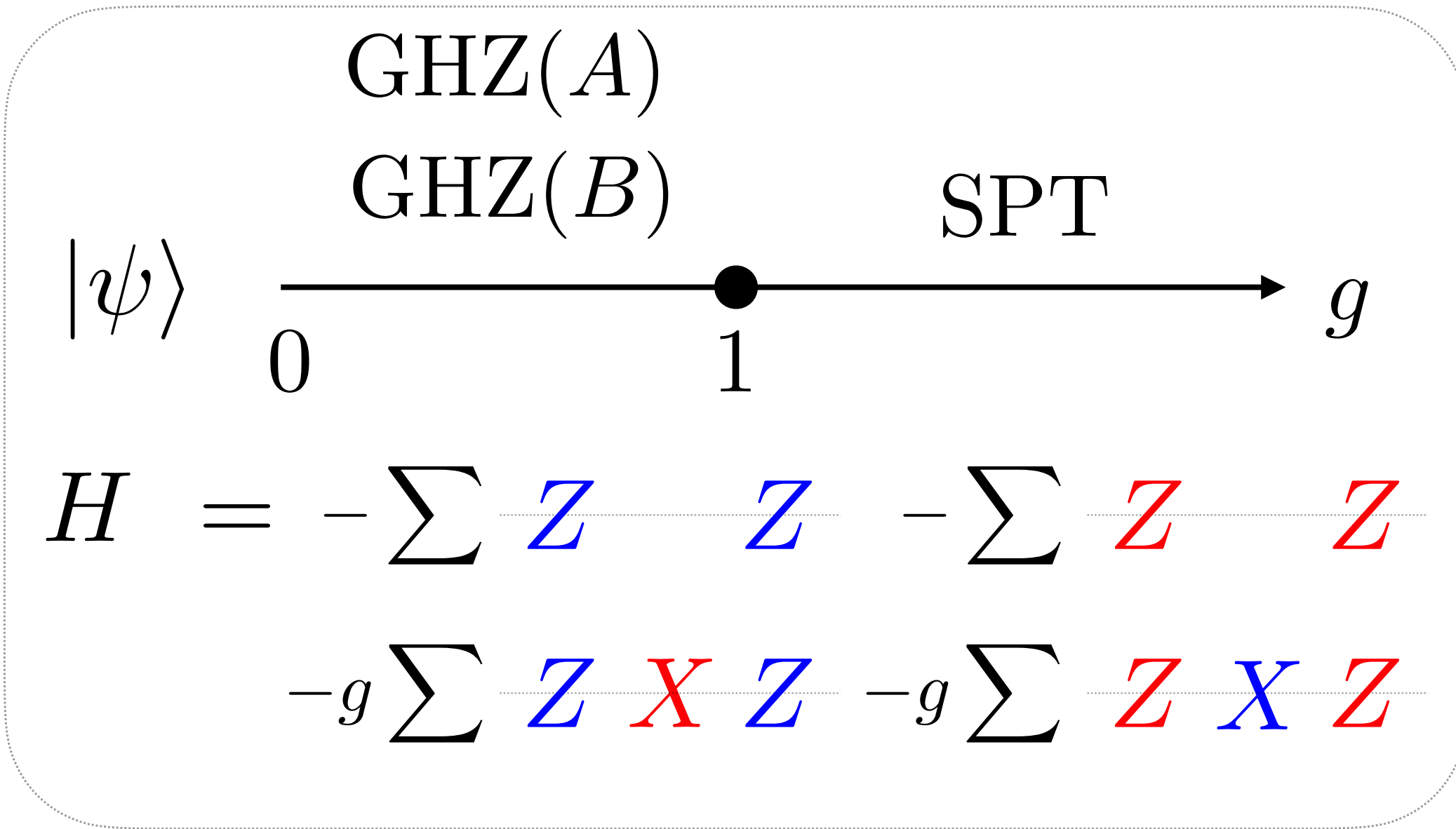


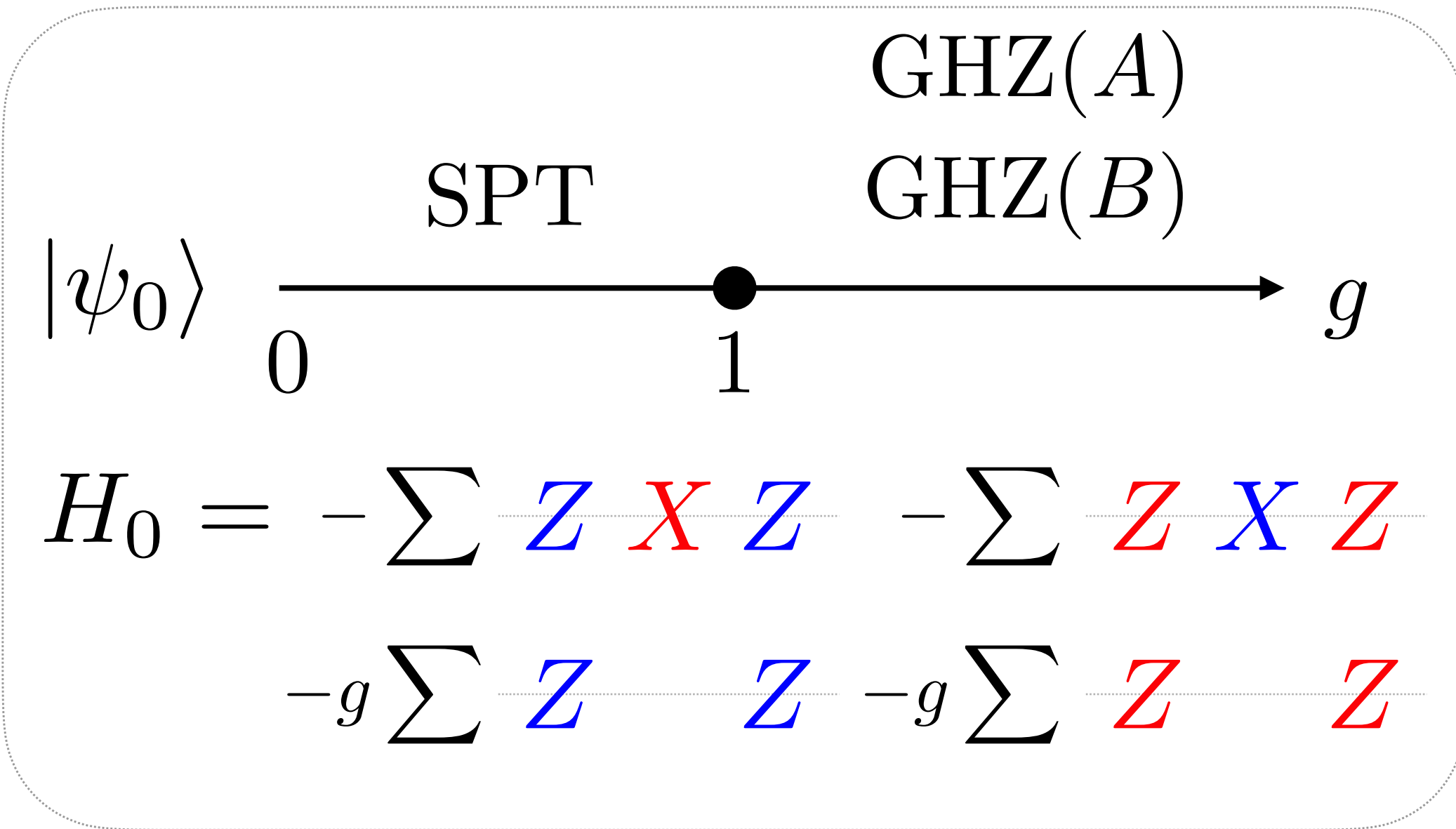
Measurement

 + Feedback



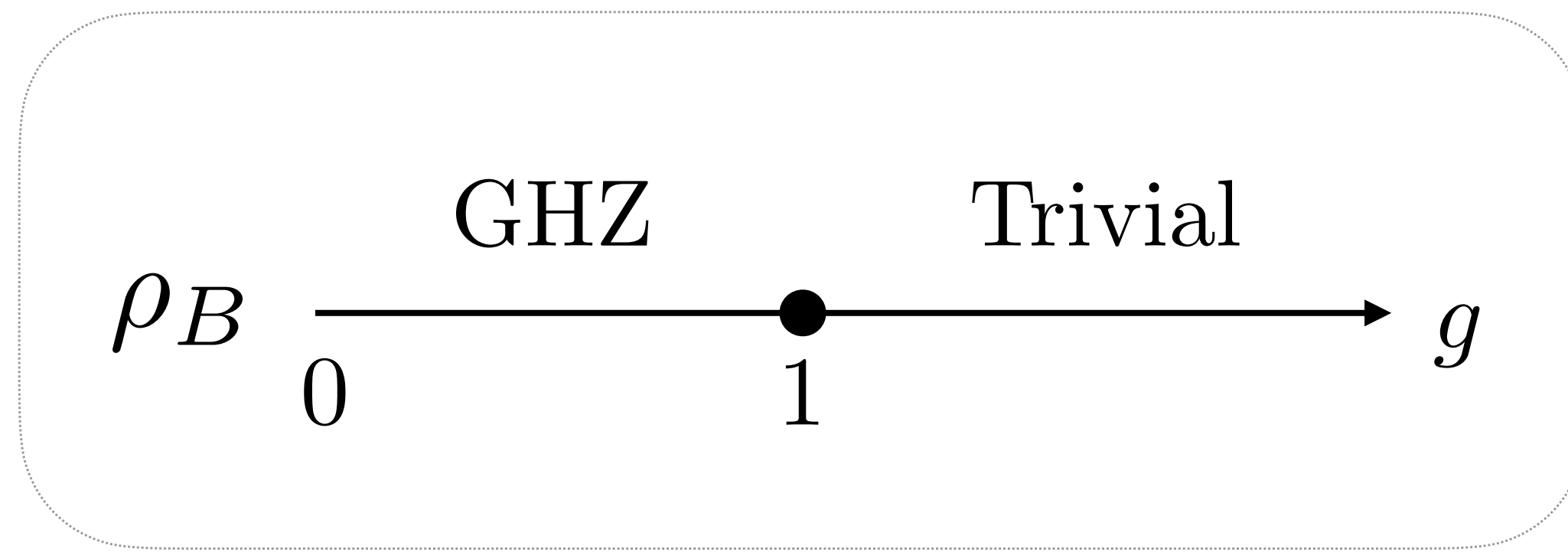
Purification 

Controlled unitary
 $U = \sum_{\alpha} U_{\alpha} P_{\alpha}$

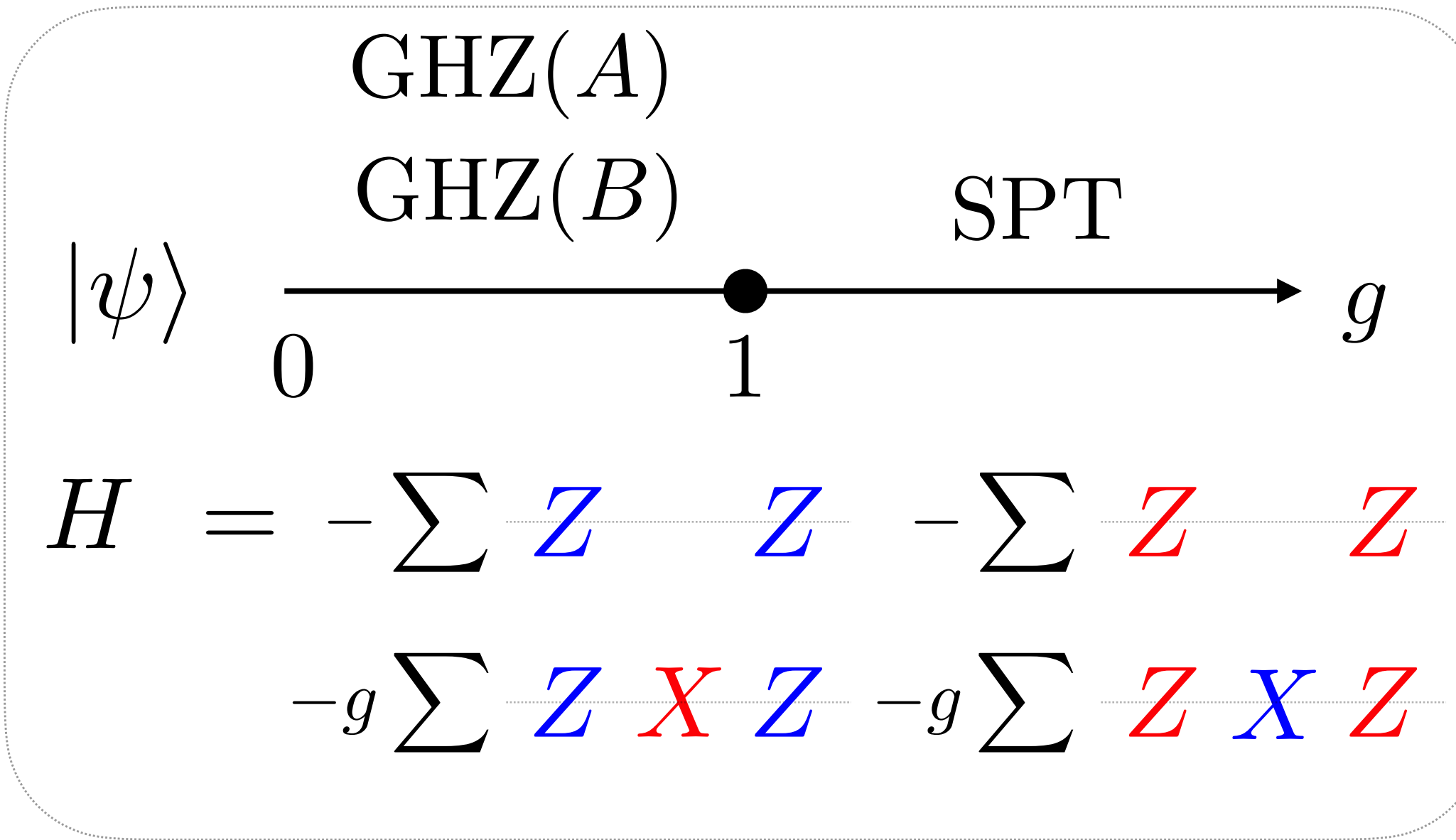





Measurement
+ Feedback



Purification



Controlled unitary

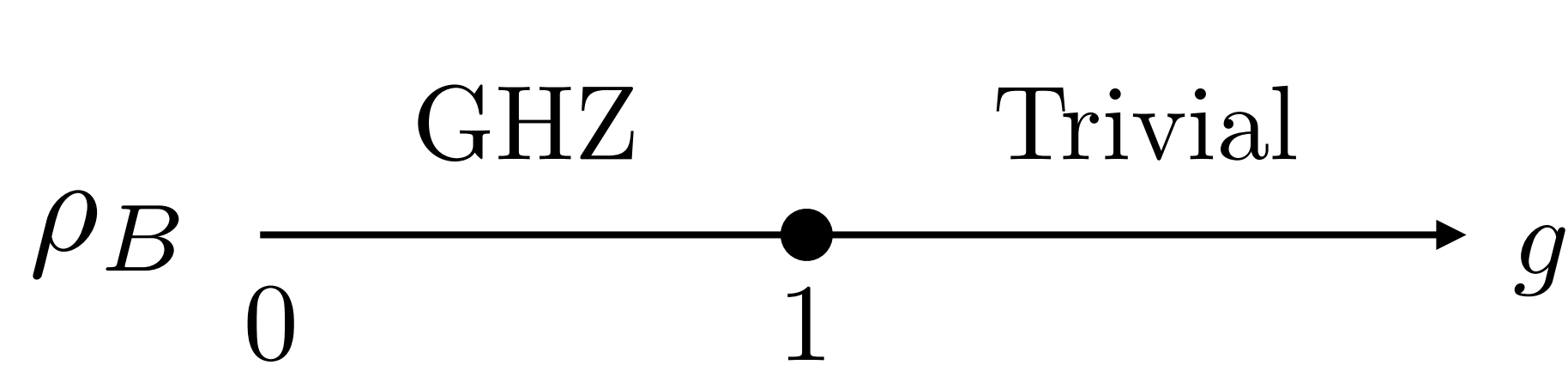
$$U = \sum_{\alpha} U_{\alpha} P_{\alpha}$$

Kennedy-Tasaki transformation (1992)

Haldane SPT \rightarrow 2 Z_2 long-range order

More recent refs: Li, Oshikawa, Zheng (2023)

Mixed-state quantum criticality ($g=1$)



Order operator

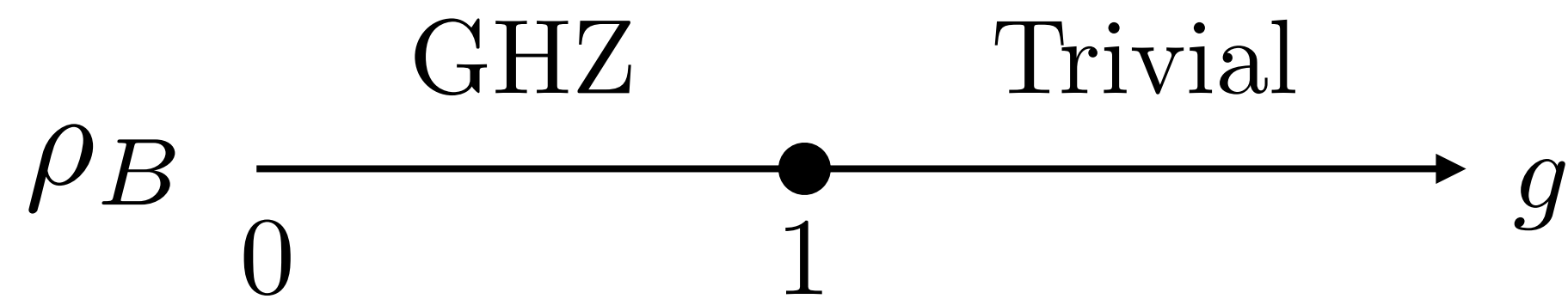
$$\langle Z_{b,i} Z_{b,j} \rangle \sim \frac{1}{|i-j|^\eta}$$

Disorder operator

$$\langle X_{b,i} X_{b,i+1} \dots X_{b,j} \rangle \sim \frac{1}{|i-j|^{2\eta}}$$

twice of Ising CFT

Mixed-state quantum criticality (g=1)



Order operator

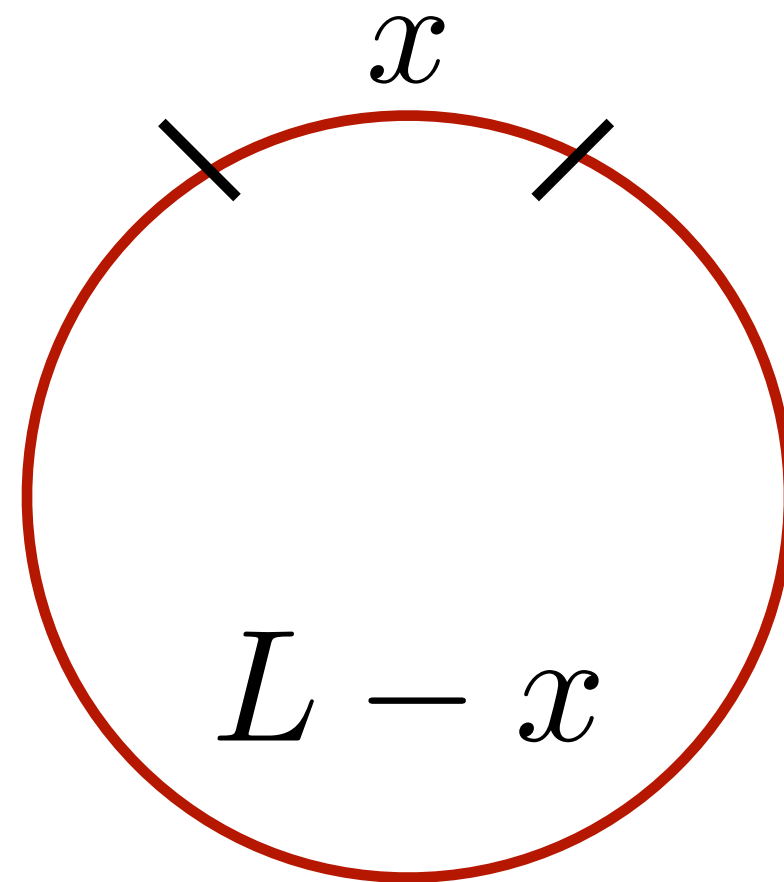
$$\langle Z_{b,i} Z_{b,j} \rangle \sim \frac{1}{|i-j|^\eta}$$

Disorder operator

$$\langle X_{b,i} X_{b,i+1} \dots X_{b,j} \rangle \sim \frac{1}{|i-j|^{2\eta}}$$

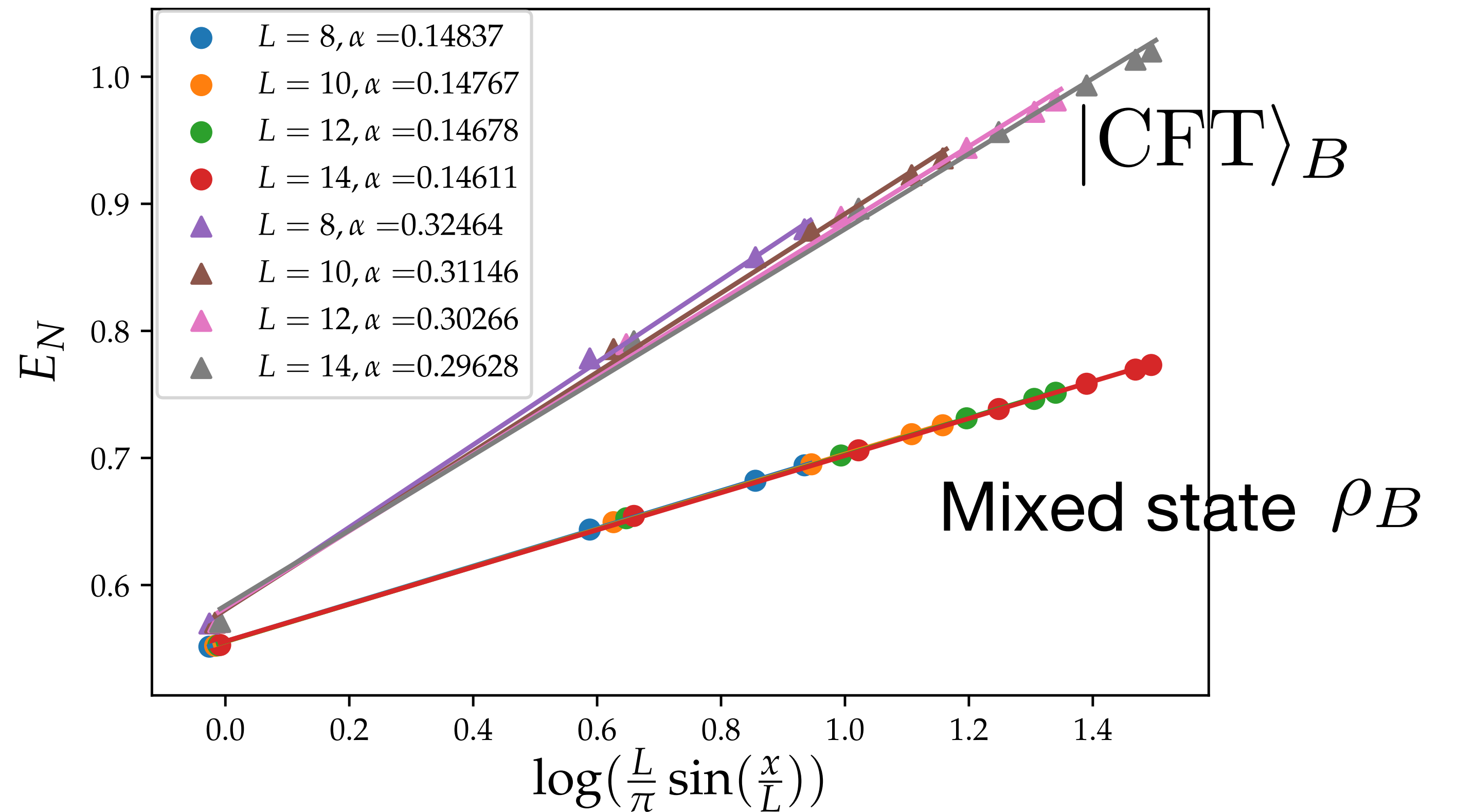
twice of Ising CFT

B sublattice



Mixed-state entanglement
(Entanglement negativity) Vidal, Werner (2001)

$$E_N(\alpha, \beta) = \alpha \log \left[\frac{L}{\pi} \sin\left(\frac{\pi x}{L}\right) \right] + \beta$$



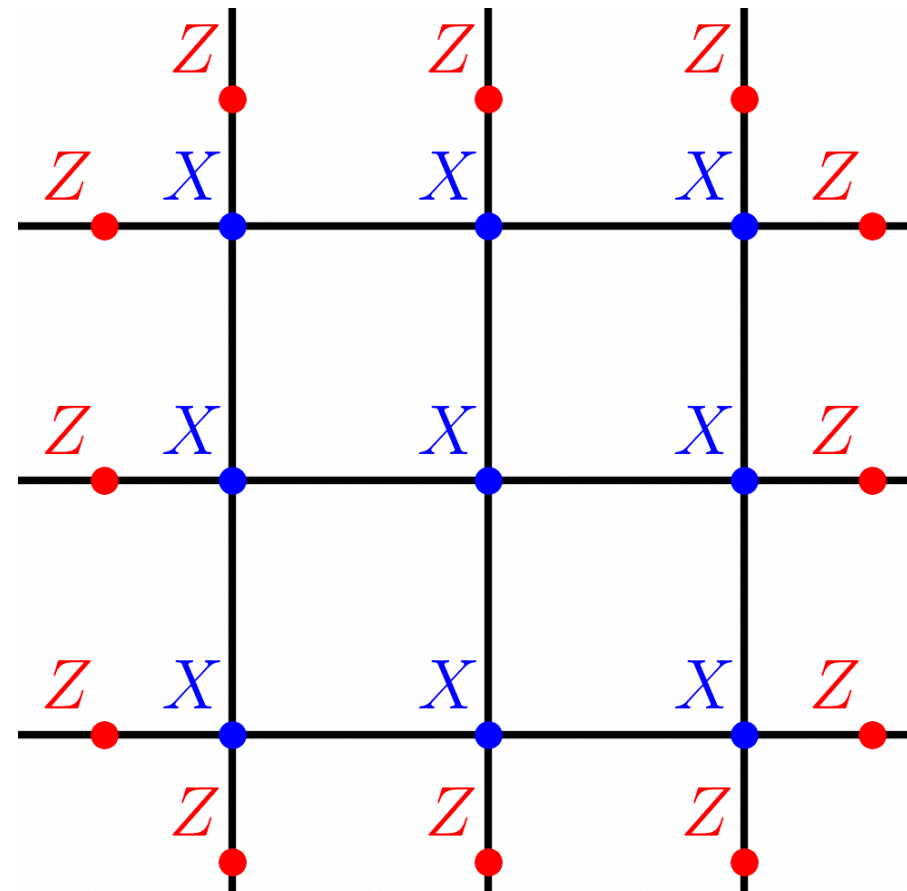
Generalization

Higher-dim.

Z_2 q-form \times Z_2 p-form **SPTs**

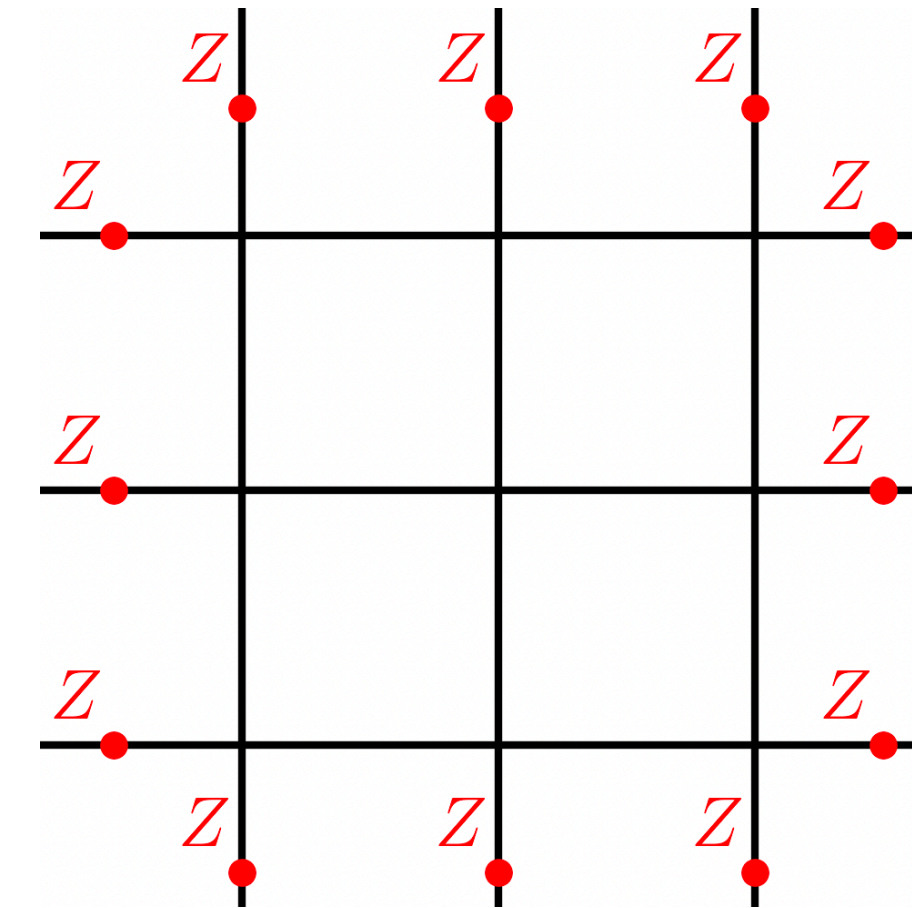


Mixed-state Z_2 **Topological Order**



Perimeter Law

$$\sim e^{-c|\tilde{\gamma}|}$$



Perimeter Law

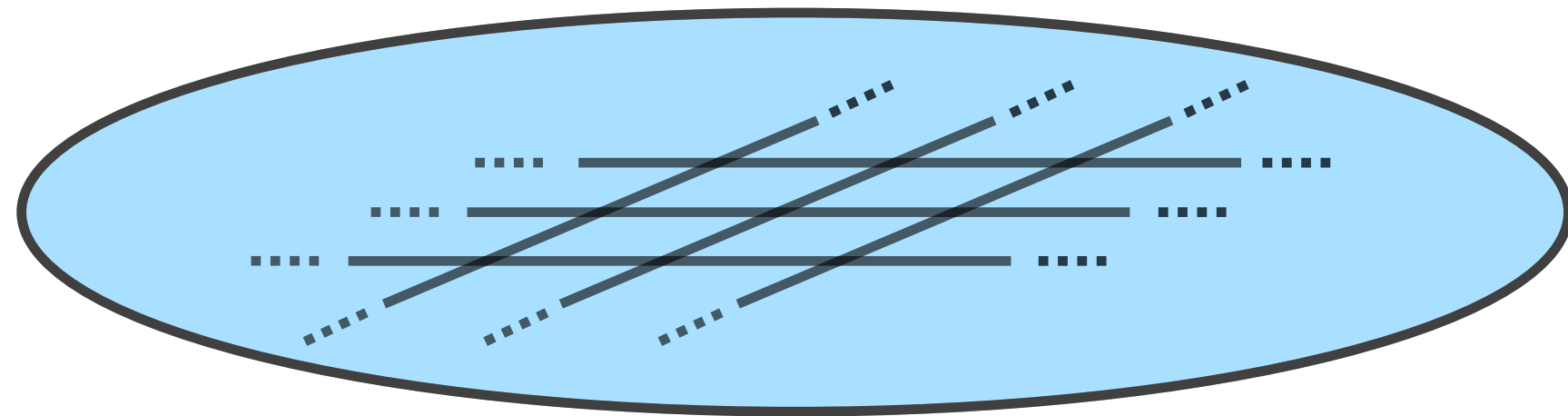
$$\sim e^{-c|\tilde{\gamma}|}$$

(Generalize to abelian $G \times H$ symmetry)

Adaptive protocol by measuring fermions

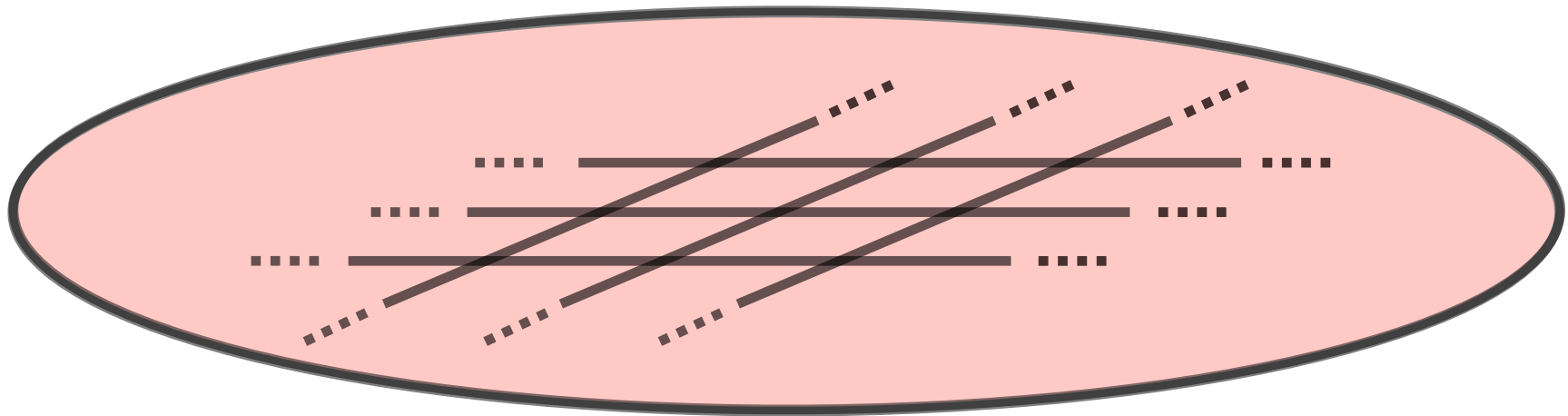
Motivated from Gutzwiller projection (1963)

$|\psi_0\rangle_\uparrow$



Free fermion (Gaussian state)

$$|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$$

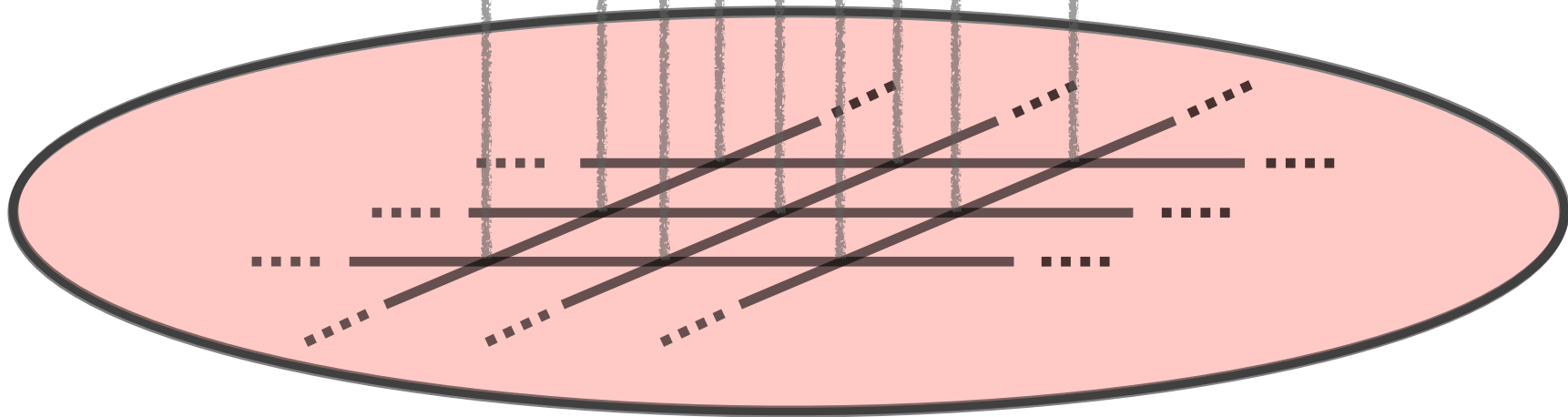
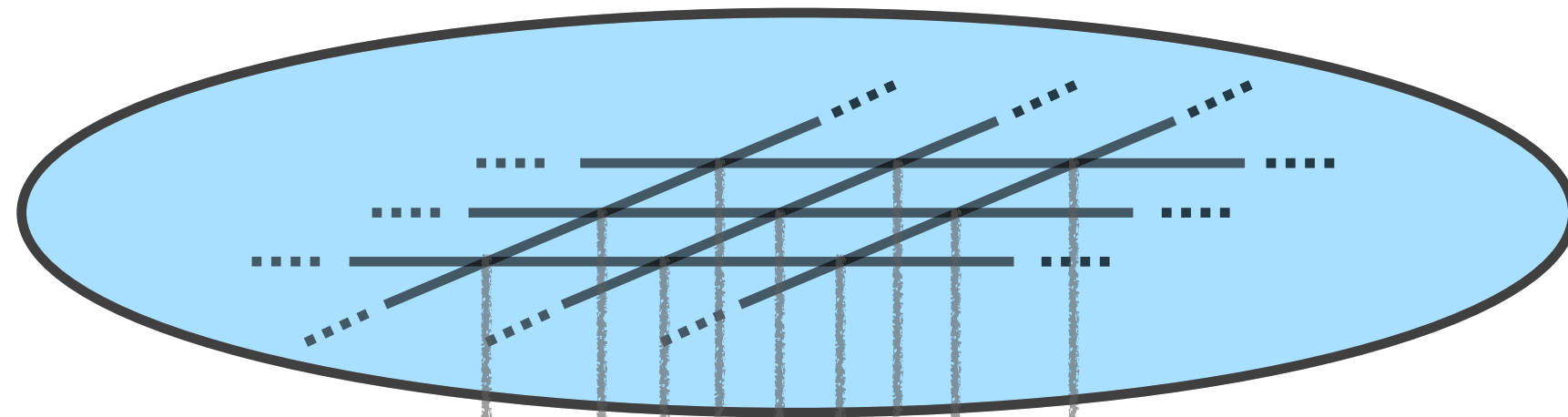


$|\psi_0\rangle_\downarrow$

Adaptive protocol by measuring fermions

Motivated from Gutzwiller projection (1963)

$|\psi_0\rangle_\uparrow$



$|\psi_0\rangle_\downarrow$

Free fermion (Gaussian state)

$$|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$$

$$|\psi_g\rangle = P_g |\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$$

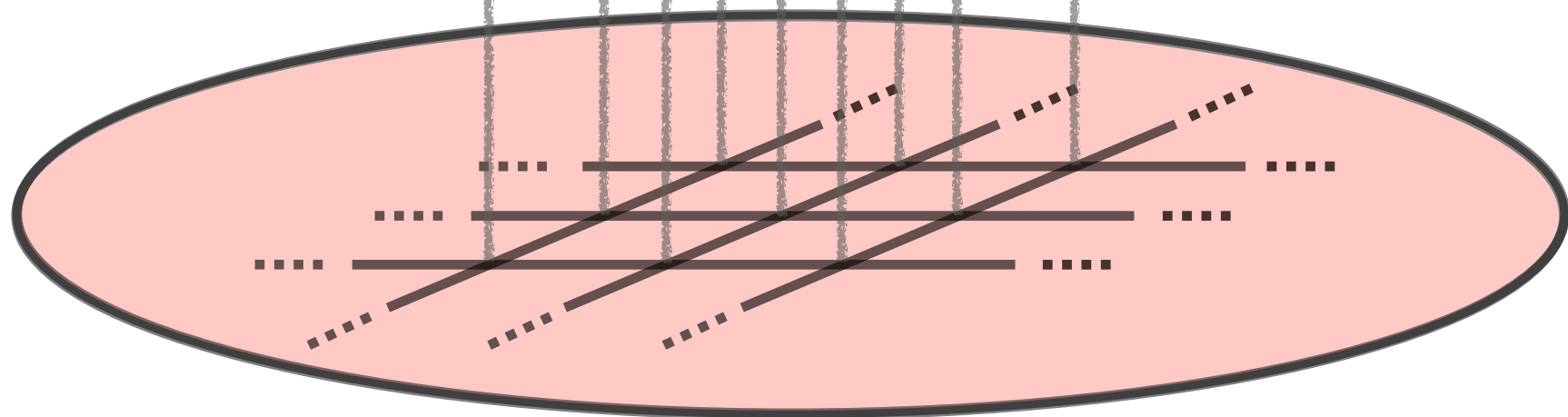
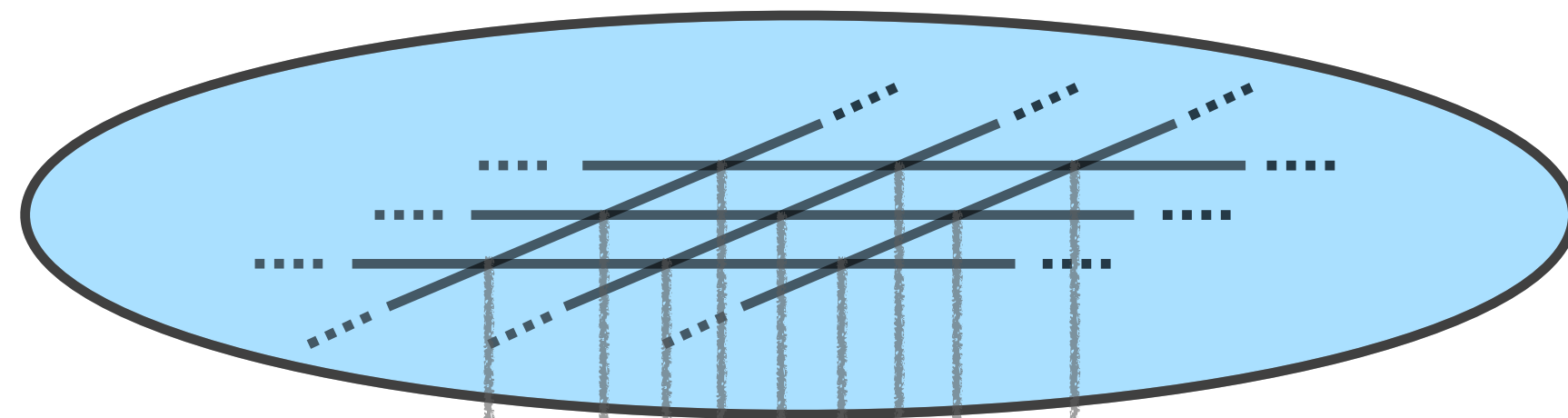


Projecting to
single-occupation per site

Adaptive protocol by measuring fermions

Motivated from Gutzwiller projection (1963)

$|\psi_0\rangle_\uparrow$



$|\psi_0\rangle_\downarrow$

Free fermion (Gaussian state)

$$|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$$

$$|\psi_g\rangle = P_g |\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$$



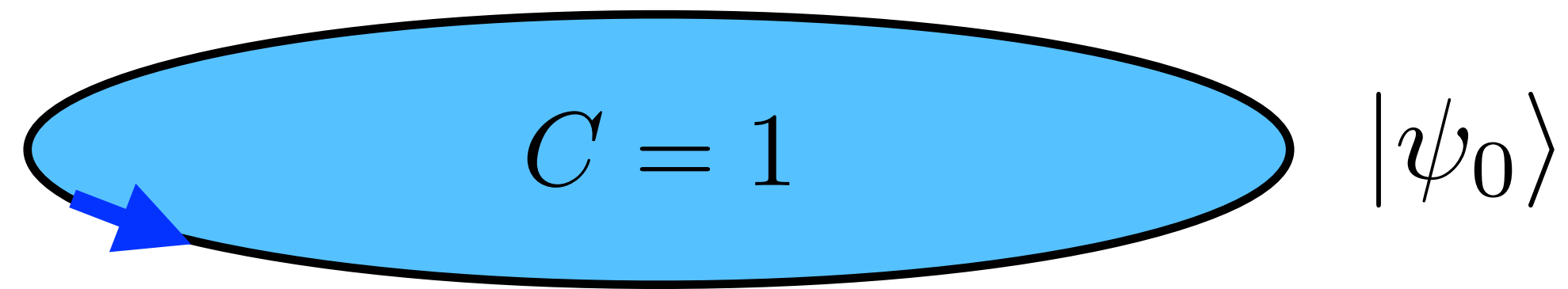
Projecting to
single-occupation per site

Gutzwiller projection by **measuring fermion number?**

Cannot control the measurement outcome

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators Review: Hasan, Kane (2010)



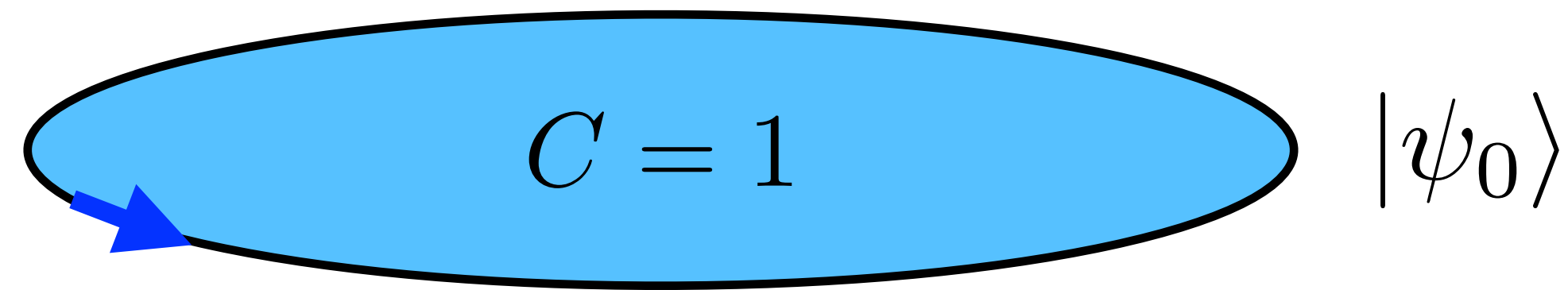
Chiral current

Trivial bulk correlation

$$\langle \psi_0 | c(\mathbf{x}) c^\dagger(\mathbf{x}') | \psi_0 \rangle \sim e^{-\frac{|\mathbf{x} - \mathbf{x}'|}{\xi}}$$

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators



Chiral current

Trivial bulk correlation

$$\langle \psi_0 | c(\mathbf{x}) c^\dagger(\mathbf{x}') | \psi_0 \rangle \sim e^{-\frac{|\mathbf{x} - \mathbf{x}'|}{\xi}}$$

Off-diagonal long-range order

Girvin, MacDonald (1987);
Kvornring, Spånslätt, Chan, Ryu (2019)

Flux attachment - > bosons

$$\tilde{c}^\dagger(\mathbf{x}) = \eta(\mathbf{x}) c^\dagger(\mathbf{x})$$

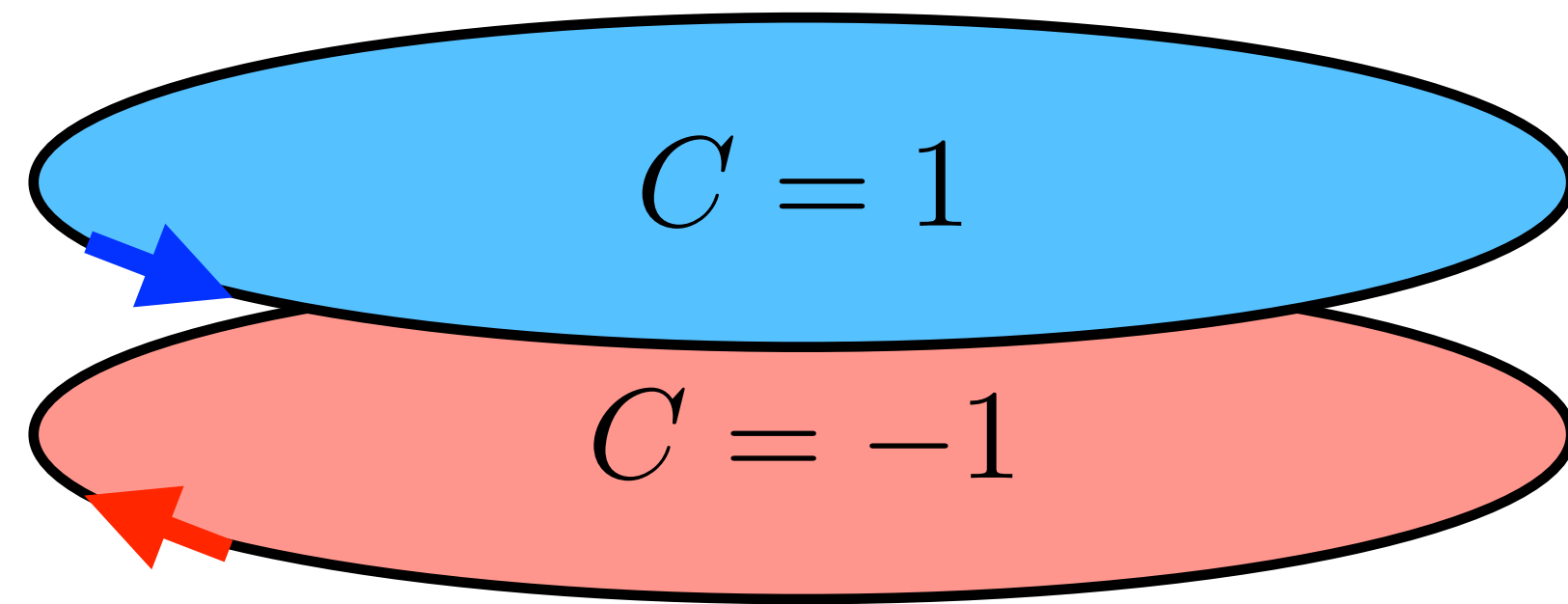
$$\eta(\mathbf{x}) = e^{i \int d^2 x' \hat{n}(\mathbf{x}') \arg(\mathbf{x} - \mathbf{x}')}$$

$$\langle \psi_0 | \tilde{c}(\mathbf{x}) \tilde{c}^\dagger(\mathbf{x}') | \psi_0 \rangle \sim \frac{1}{|\mathbf{x} - \mathbf{x}'|^\alpha}$$

Flux attachment from measurement + feedback !

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators $|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$



$|\psi_0\rangle_\uparrow$

$$S^+(\mathbf{x}) = c_\uparrow^\dagger(\mathbf{x})c_\downarrow(\mathbf{x})$$

$|\psi_0\rangle_\downarrow$

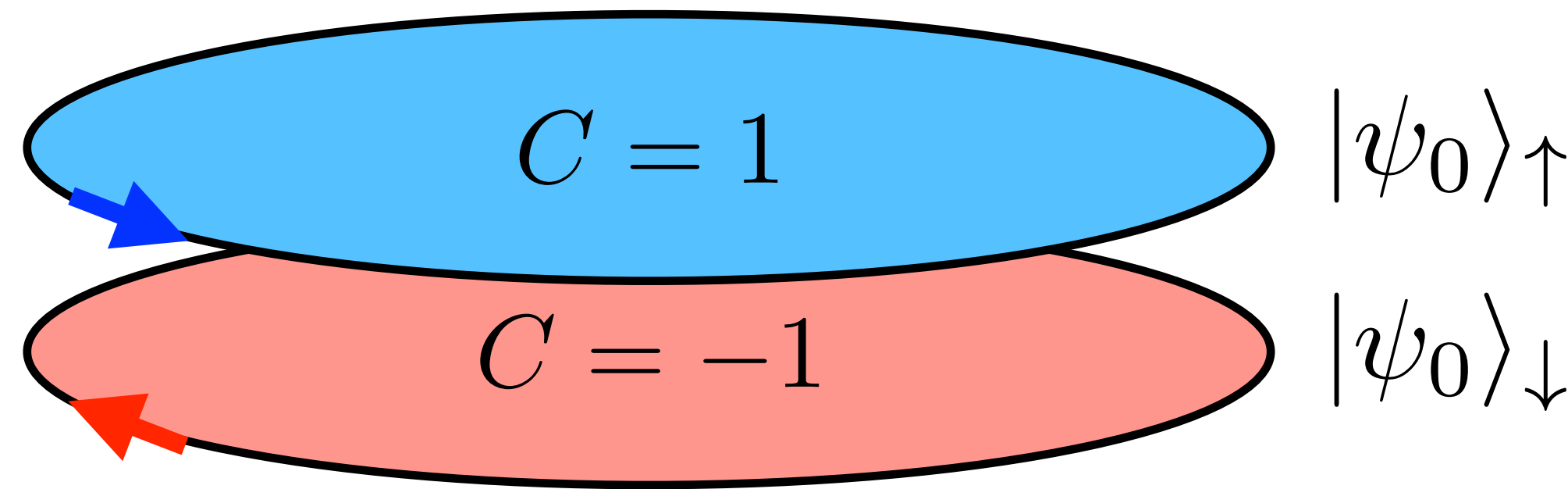
$$\hat{\phi}_\uparrow(\mathbf{x}) = \int d^2x' \hat{n}_\uparrow(\mathbf{x}') \arg(\mathbf{x} - \mathbf{x}')$$

$$\hat{\phi}_\downarrow(\mathbf{x}) = - \int d^2x' \hat{n}_\downarrow(\mathbf{x}') \arg(\mathbf{x} - \mathbf{x}')$$

$$\begin{aligned} & \langle \psi_0 | \underline{e^{i[\hat{\phi}_\uparrow(\mathbf{x}) - \hat{\phi}_\downarrow(\mathbf{x})]} S^+(\mathbf{x}) S^-(\mathbf{x}') e^{-i[\hat{\phi}_\uparrow(\mathbf{x}') - \hat{\phi}_\downarrow(\mathbf{x}')]} | \psi_0 \rangle} \\ & \sim \langle \psi_0 |_\uparrow \tilde{c}_\uparrow(\mathbf{x}') \tilde{c}_\uparrow^\dagger(\mathbf{x}) | \psi_0 \rangle_\uparrow \langle \psi_0 |_\downarrow \tilde{c}_\downarrow(\mathbf{x}) \tilde{c}_\downarrow^\dagger(\mathbf{x}') | \psi_0 \rangle_\downarrow \sim \frac{1}{|\mathbf{x} - \mathbf{x}'|^{2\alpha}} \end{aligned}$$

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators $|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$



$|\psi_0\rangle_\uparrow$

$|\psi_0\rangle_\downarrow$

(1) measuring onsite fermion number

(2) unitary $n = \{n_1, n_2, \dots\}$

$$U_n = \prod_{\mathbf{x}} e^{-i\phi(\mathbf{x})S^z(\mathbf{x})}$$

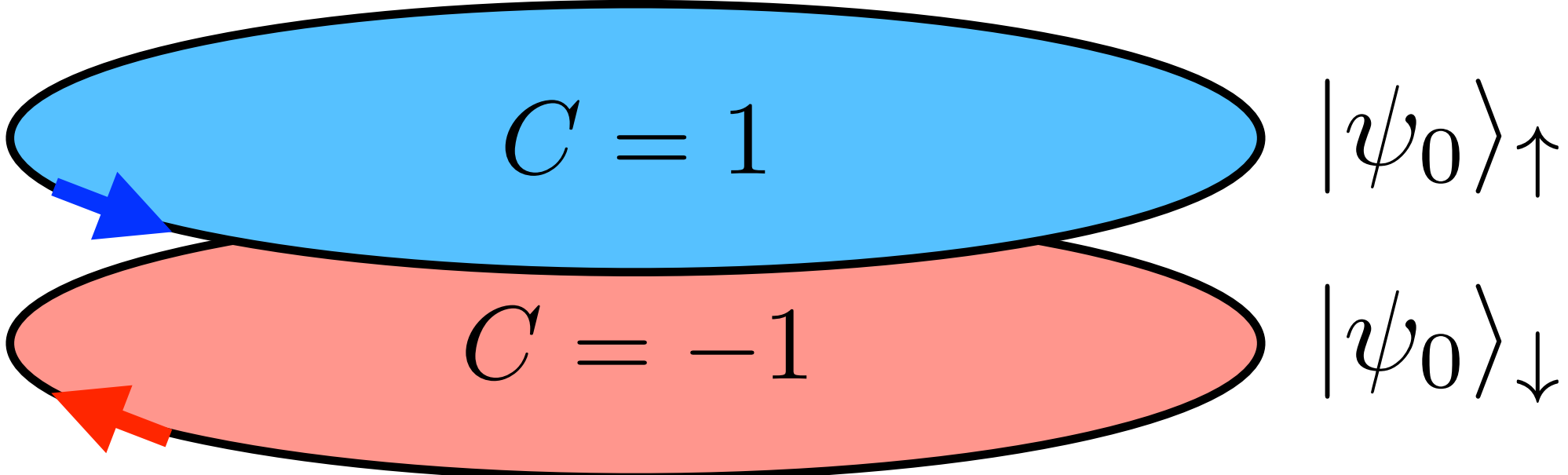
$$\phi(\mathbf{x}) = \sum_{\mathbf{x}'} n(\mathbf{x}') \arg(\mathbf{x} - \mathbf{x}')$$

$$U_n^\dagger S^+(\mathbf{x}) U_n = e^{i[\phi_\uparrow(\mathbf{x}) - \phi_\downarrow(\mathbf{x})]} S^+(\mathbf{x})$$

$$U_n^\dagger S^-(\mathbf{x}') U_n = e^{-i[\phi_\uparrow(\mathbf{x}') - \phi_\downarrow(\mathbf{x}')]} S^-(\mathbf{x}')$$

Mixed-state quantum criticality from gapped states of matter

2d Chern insulators $|\psi_{0,\uparrow}\rangle \otimes |\psi_{0,\downarrow}\rangle$



- (1) measuring onsite fermion number
- (2) unitary $n = \{n_1, n_2, \dots\}$

$$U_n = \prod_{\mathbf{x}} e^{-i\phi(\mathbf{x})S^z(\mathbf{x})}$$

$$\phi(\mathbf{x}) = \sum_{\mathbf{x}'} n(\mathbf{x}') \arg(\mathbf{x} - \mathbf{x}')$$

$$U_n^\dagger S^+(\mathbf{x}) U_n = e^{i[\phi_\uparrow(\mathbf{x}) - \phi_\downarrow(\mathbf{x})]} S^+(\mathbf{x})$$

$$U_n^\dagger S^-(\mathbf{x}') U_n = e^{-i[\phi_\uparrow(\mathbf{x}') - \phi_\downarrow(\mathbf{x}')]} S^-(\mathbf{x}')$$

$$e^{i[\hat{\phi}_\uparrow(\mathbf{x}) - \hat{\phi}_\downarrow(\mathbf{x})]} S^+(\mathbf{x}) S^-(\mathbf{x}') e^{-i[\hat{\phi}_\uparrow(\mathbf{x}') - \hat{\phi}_\downarrow(\mathbf{x}')]}$$



$$S^+(\mathbf{x}) S^-(\mathbf{x}') \sim \frac{1}{|\mathbf{x} - \mathbf{x}'|^{2\alpha}}$$

Summary - adaptive protocol

Measurement + feedback unitary + classical communication

—> efficient manipulation of many-body states

Low-depth realization of mixed-state order and criticality

Pure		Mixed
$Z_2 \times Z_2$ SPT	— — —>	Long-range (topological) order
Gapped Chern insulator (exp. decaying correlation)	— — —>	Mixed-state quantum criticality (algebraic correlation)
1d fermion (algebraic correlation)	— — —>	1d fermion (enhanced algebraic correlation)

Summary - adaptive protocol

Measurement + feedback unitary + classical communication

—> efficient manipulation of many-body states

Low-depth realization of mixed-state order and criticality

Open questions

(1) Non-local transformation & flux attachment

More non-trivial orders/criticality?

(2) Limitations of measurement + feedback

(3) Mixed-state phases of matter - characterization & diagnostic